



## Estimating Turbulence Statistics and Parameters from Lidar Measurements. Remote Sensing Summer School

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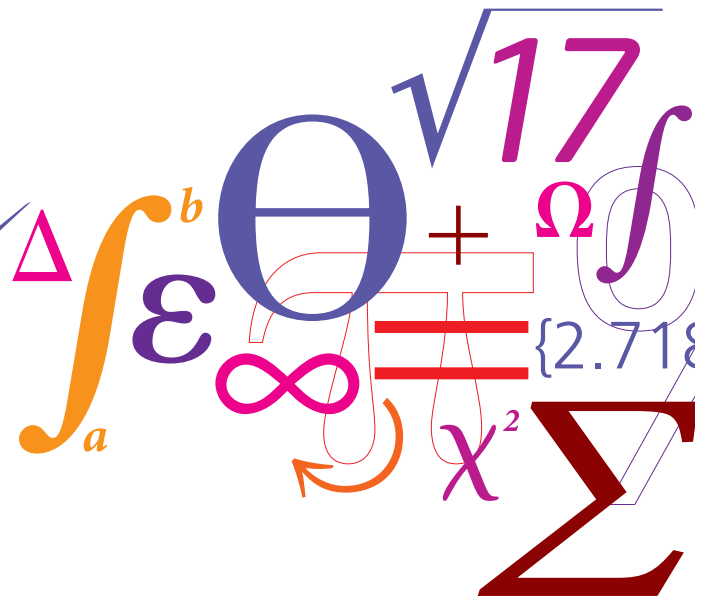
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# Estimating Turbulence Statistics and Parameters from Lidar Measurements

## Remote Sensing Summer School

Department of  
Wind Energy  
E Report 2015

$$P = \frac{1}{2} \rho A v^3 C_p$$



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June 2015

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**Title:** Estimating Turbulence Statistics and Parameters from Lidar Measurements

**Department:** DTU Wind Energy

**Summary:** This report is prepared as a written contribution to the Remote Sensing Summer School, that is organized by the Department of Wind Energy, Technical University of Denmark. It provides an overview of the state-of-the-art with regards to estimating turbulence statistics from lidar measurements, as well as experimental evidence from different measurement campaigns at a test center in Denmark. Several measurement configurations from the commercial and research lidars are described along with mathematical formulations of estimated turbulence statistics and parameters for the respective configuration. The so-called velocity Azimuth Display (VAD) and the Doppler Beam Swinging (DBS) methods of post processing the lidar data are investigated in greater details, partly due to their wide use in commercial lidars. It is demonstrated that the VAD or DBS techniques result in introducing significant systematic errors in the estimated turbulence statistics. New techniques of post-processing the lidar measurements are also discussed, amongst others the so-called six-beam technique, which reduces the systematic errors in the estimated turbulence statistics significantly. The report ends with recommendations for careful handling of lidar data for quantifying turbulence along with some future perspectives.

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# Preface

The goal of this report is to provide an overview of the state-of-the-art with regards to estimating turbulence statistics from lidar measurements, as well as providing experimental evidence from different measurement campaigns at a test center in Denmark. Objectively the attempt is to provide the readers with answers to two questions:

1. With the existing technology and knowledge, can we routinely use wind lidars to estimate turbulence statistics?
2. What improvements in the existing lidar technology and knowledge are required in order to estimate turbulence statistics, where the degree of accuracy and precision is comparable to that estimated using the traditional meteorological mast anemometry?

As far as possible the report is structured such that each chapter is independent of each other (except chapter 1), where a reader that is interested in only a specific topic can skip other chapters and simply jump to the concerned chapter without much loss of information.

A report on turbulence measurements is impossible without providing some basic mathematical terminology. Therefore in chapter 1, basic turbulence terminology is defined that is used in the rest of the report. It is highly recommended that the reader goes through the definitions and familiarizes with the notations in this chapter, since they will be frequently used in the remaining chapters. A reader that is already familiar with the basic terminology can simply skip this chapter and refer to the nomenclature provided at the beginning of this report.

Chapter 2 begins with the motivation of using lidars to estimate turbulence statistics, followed by some basic of coherent Doppler lidars with regards to two different technologies, namely the continuous wave and the pulsed lidars. A brief description of the turbulence statistics that is relevant for wind energy is also provided.

In chapter 3, illustrations of different measurement configurations are provided, where further division is made between the commercial and research configurations. Some of these illustrations are referred to in chapters 4 and 5, where the reader who has not gone through the description of these configurations can simply refer to the respective figures. Additionally for some of the measurement configurations, mathematical formulations are provided for different turbulence parameters. Although a bit complicated, we believe that this will further help the readers in understanding what is possible with the respective measurement configurations in terms of estimating turbulence parameters, even if the readers simply glance through the mathematical formulations without going into details.

Chapter 4 provides a review of the state-of-the-art with regards to estimating turbulence statistics using lidar measurements. The review is performed by grouping the turbulence statistics, where the definitions introduced in chapter 1 are used.

In chapter 5, results from different measurement campaigns at a test center in Denmark are provided. The chapter is divided such that detailed information of each measurement campaign is given as individual section, and as far as possible uniformity in describing the measurement campaigns is maintained.

The report ends with chapter 6, where an attempt is made to answer the aforementioned questions. A section on future perspectives specifically tackles the second question stated above.

**June, 2015**

**Place:** Roskilde, Denmark

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# Nomenclature

$C \approx 1.5$	universal Kolmogorov constant
$C_2 \approx 2$	Kolmogorov constant related to $D_{11}(r_1)$
$D_{ij}(\mathbf{r})$	velocity structure function
$F_{ij}(k_1)$	one-dimensional velocity spectrum
$I$	longitudinal turbulence intensity
$L_p$	range gate length ( $c\tau/2$ )
$R_{ij}(\mathbf{r})$	cross covariance function
$\mathbf{R} = \mathbf{R}(0)$	covariance matrix
$\mathbf{k}$	wave vector in the Fourier domain
$\mathbf{n}$	unit directional vector
$\mathbf{r}$	separation vector in three dimensions
$\mathbf{x}$	position vector in three dimensions
$\text{coh}_{ij}(k_1)$	coherence function
$\langle S(v_r) \rangle$	mean Doppler spectra
$\langle u \rangle$	mean wind speed
$\langle u'^2 \rangle$	variance of the $u$ component
$\langle u'v' \rangle$	covariance between the $u$ and $v$ components
$\langle u'w' \rangle$	covariance between the $u$ and $w$ components
$\langle v_r'^2 \rangle$	radial velocity variance
$\langle v'^2 \rangle$	variance of the $v$ component
$\langle v'w' \rangle$	covariance between the $v$ and $w$ components
$\langle w'^2 \rangle$	variance of the $w$ component
$\langle w'^3 \rangle$	third moment of the vertical velocity
$\mathbf{v}$	wind field vector
$\tilde{D}(\delta)$	filtered radial velocity structure function for a separation distance $d_f\delta$
$\tilde{D}(r)$	filtered radial velocity structure function for a separation distance $r$
$\tilde{D}(r_1)$	filtered radial velocity structure function for a separation distance $r_1$
$\tilde{F}(k_1)$	filtered radial velocity spectrum
$\tilde{R}(r)$	filtered covariance function of the radial velocity for a separation distance $r$
$c$	speed of light
$d_f$	focus distance for a C-W lidar and center of the range gate for a pulsed lidar
$i, j$	indices that take values 1, ., 3 and denote the component of the wind field
$k_1, k_2, k_3$	components of the wave vector along the $x_1, x_2, x_3$ axes respectively
$l$	Rayleigh length
$r$	separation distance along the lidar beam
$r_1, r_2, r_3$	separation distances along the $x_1, x_2, x_3$ axes respectively
$r_b$	lidar beam radius
$u$	longitudinal component of the wind field in the $x_1$ direction
$v$	transversal component of the wind field in the $x_2$ direction
$v_r$	radial velocity
$w$	vertical component of the wind field in the $x_3$ direction
$w_p$	pulse width
$x_1, x_2, x_3$	axes defining the right handed cartesian coordinate system
$z$	height above the ground
$\mathcal{L}$	outer length scale of turbulence
$\Phi_{ij}(\mathbf{k})$	three-dimensional spectral velocity tensor
$\beta$	angle between the lidar beam and the mean wind

$\Theta$	mean wind direction
$\alpha$	elevation angle
$\chi_{ij}(k_1, r_2, r_3)$	cross spectra at separation distances $r_2$ and $r_3$
$\delta$	angle subtended by two lidar beams in a VAD scanning mode
$\ell_{ij}$	integral length scale
$\langle \sigma_s^2 \rangle$	second central moment of the Doppler spectrum (Doppler spectrum width)
$\phi$	zenith angle
$g$	acceleration due to gravity
$L$	Obukhov Length
$u_*$	friction velocity
$\theta_v$	virtual potential temperature
$\overline{w'\theta'_v}$	surface virtual kinematic heat flux
$\kappa$	von Kármán constant
$\tau$	pulse duration
$\lambda_b$	wavelength of the emitted radiation
$\theta$	azimuth angle
$\varepsilon$	energy dissipation rate
RHI	range height indicator
VAD	velocity azimuth display
CW	continuous-wave

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# Chapter 1

## Mathematical Preliminaries

The main purpose of writing this chapter in the beginning is that the notations defined here will be helpful in understanding the following chapters. Discussing turbulence without introducing the notations would simply make the text in the remaining chapters quite vague and cumbersome to read. Therefore we have included some mathematics using a uniform set of notations, in order to provide a clear perspective. However, It is to be noted that only in section 2.1.2, the notations deviate slightly from the rest of the report. This is because the content of section 2.1.2 is directly taken from Vasiljevic [2014]. Nevertheless the slight changes in the notations in section 2.1.2 does not have any influence on the readability of the rest of the report.

Turbulence statistics are usually described in a some standard coordinate system. If the selection of a coordinate system is left at the prerogative of the scientist or an engineer performing the measurement, then interpreting turbulence statistics would be very cumbersome. Fortunately in the meteorological world, a consensus has been achieved where the wind field components are described in a coordinate system such that one of its components is in the mean wind direction. In some literature [Wilczak et al., 2001], such a coordinate system is also called as the streamline coordinate system. Therefore at first we define the base coordinate system to be right-handed as shown in Fig. 1.1, where the  $x_1$  axis can be considered to be pointing east, the  $x_2$  axis can be considered to be pointing north, and  $x_3$  axis can be considered to be pointing vertically upwards. The mean wind direction is shown to make a positive angle  $\Theta$  with respect to the  $x_2$  axis, i.e. north in the clockwise direction such that a wind direction of  $0^\circ$  denotes the wind blowing from north to south. Measurements of components of the wind field are usually carried out in some arbitrary base coordinate system. It is therefore necessary to perform coordinate transformation on the measured wind field components in

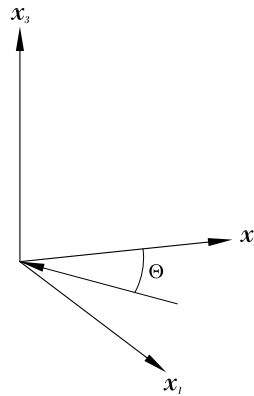


Figure 1.1: Standard meteorological convention of depicting the mean wind direction

the some arbitrary base coordinate system such that the final coordinate system is aligned with the streamline coordinate system. For simplicity in the rest of the report, it is assumed that the streamline coordinate system is aligned with the coordinate system of the lidar such that  $u$  is along the positive  $x_1$  axis.

In this report we will often switch between the bold faced vector notation and the Einstein indicial notation. We define the wind field as  $\mathbf{v} = (u, v, w)$ , such that  $u$  (longitudinal component) is in the mean wind direction,  $v$  (transversal component) is perpendicular (in a right-handed system) to the mean wind direction, and  $w$  is in the vertical  $x_3$  direction. If we consider that the fluctuations of the wind field are homogeneous in space then the auto or cross covariance functions can be defined only in terms of the separation distance as,

$$R_{ij}(\mathbf{r}) = \langle v'_i(\mathbf{x}) v'_j(\mathbf{x} + \mathbf{r}) \rangle, \quad (1.1)$$

where  $R_{ij}(\mathbf{r})$  is the auto or cross correlation function,  $i, j = (1, 2, 3)$  are the indices corresponding to the components of the wind field,  $\mathbf{x}$  is the position vector in the three dimensional Cartesian coordinate system,  $\mathbf{r} = (r_1, r_2, r_3)$  is the separation vector,  $\langle \rangle$  denotes ensemble averaging, and  $'$  denotes fluctuations about the ensemble average. Eq. (1.1) denotes a two-point turbulent statistic. At  $\mathbf{r} = 0$  we then get a single-point turbulent statistic, which we can denote as the variances and covariances. In matrix form it can be written as,

$$\mathbf{R} = \begin{bmatrix} \langle u'^2 \rangle & \langle u'v' \rangle & \langle u'w' \rangle \\ \langle v'u' \rangle & \langle v'^2 \rangle & \langle v'w' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'^2 \rangle \end{bmatrix}, \quad (1.2)$$

where the diagonal terms are the variances of the respective wind field components and the off-diagonal terms are the covariances. Here, it is implied that  $\mathbf{R} = \mathbf{R}(0)$ , and we drop the argument and the bracket for simplicity. In wind energy, one frequently used statistic is the turbulence intensity, which according to the IEC [2005] standards is defined as,

$$I = \frac{\sigma_u}{\langle u \rangle}, \quad (1.3)$$

where  $I$  is the turbulence intensity, and  $\sigma_u = \sqrt{\langle u'^2 \rangle}$  is the standard deviation of the horizontal (or the longitudinal component) wind speed. From the definition of  $\mathbf{R}(\mathbf{r})$  and  $\mathbf{R}$ , we can define integral length scale as,

$$\ell_{ij} = \frac{1}{R_{ij}} \int_0^\infty R_{ij}(r_1) dr_1, \quad (1.4)$$

Similar to  $R_{ij}(\mathbf{r})$ , another useful two-point statistic to characterize turbulence is the velocity structure function, which is defined as,

$$D_{ij}(\mathbf{r}) = \langle (v'_i(\mathbf{x} + \mathbf{r}) - v'_i(\mathbf{x})) (v'_j(\mathbf{x} + \mathbf{r}) - v'_j(\mathbf{x})) \rangle. \quad (1.5)$$

On many occasions it convenient to study turbulence in the Fourier domain instead of the time domain. To this extent, we can define the spectral velocity tensor (or the three-dimensional spectral density) as the Fourier transform of  $R_{ij}(\mathbf{r})$ ,

$$\Phi_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int R_{ij}(\mathbf{r}) \exp(i \mathbf{k} \cdot \mathbf{r}) d\mathbf{r}, \quad (1.6)$$

where  $\Phi_{ij}(\mathbf{k})$  is the three-dimensional spectral velocity tensor,  $\mathbf{k} = (k_1, k_2, k_3)$  is the wave vector, and  $\int d\mathbf{k} = \int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty dk_1 dk_2 dk_3$ . From Eq. (1.6) it is obvious that  $R_{ij}(\mathbf{r})$  is the inverse Fourier transform of  $\Phi_{ij}(\mathbf{k})$ . A single-point statistic is then given as,

$$R_{ij} = \int \Phi_{ij}(\mathbf{k}) d\mathbf{k}. \quad (1.7)$$

Practically, it is not possible to measure a spectral velocity tensor, since we would need measurements at all points in a three-dimensional space. A one-dimensional velocity spectrum is then used, which is defined as,

$$F_{ij}(k_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ij}(r_1) \exp(-ik_1 r_1) dr_1 \quad (1.8)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ij}(\mathbf{k}) dk_3 dk_2. \quad (1.9)$$

Alternatively, one can resort to using a semi-empirical turbulence model [Mann, 1994]. In this model, the turbulence structure in the neutral atmospheric surface layer is characterized by  $\Phi_{ij}(\mathbf{k})$ , which is quantified as a function of only three parameters,  $C\varepsilon^{2/3}$ , a product of the universal Kolmogorov constant  $C \approx 1.5$  [Pope, 2000] and the turbulent kinetic energy dissipation rate to the two-third power  $\varepsilon^{2/3}$ , the outer length scale of turbulence  $\mathcal{L}$ , which is the length scale corresponding to the maximum spectral energy, and an anisotropy parameter. Another important statistic in the Fourier domain is the coherence function defined as,

$$\text{coh}_{ij}(k_1) = \frac{|\chi_{ij}(k_1, r_2, r_3)|^2}{F_{ii}(k_1)F_{jj}(k_1)}, \quad (1.10)$$

where  $\chi_{ij}(k_1, r_2, r_3)$  denotes the cross spectra between the components  $i$  and  $j$ , and  $F_{ii}(k_1) = \chi_{ii}(k_1, 0, 0)$ ,  $F_{jj}(k_1) = \chi_{jj}(k_1, 0, 0)$  (no summation over repeated indices) are the one-dimensional spectra of the  $i$  and  $j$  components respectively.

In turbulence studies it is common to classify the observations based on atmospheric stability, which denotes production/dampning of turbulence due to heating/cooling of the surface. There are several ways to characterize atmospheric stability mathematically. In this report we characterize it by Obukhov length  $L$ , which can be physically interpreted as the height at which the mechanical rate of production of turbulence (due to friction at the surface) become equal to that of the buoyant rate of production of turbulence [Wyngaard, 2010]. Mathematically it is given as,

$$L = -\frac{u_*^3 \theta_v}{\kappa g \overline{w' \theta_v'}}, \quad (1.11)$$

where  $u_*$  is the friction velocity,  $\kappa = 0.4$  is the von Kármán constant,  $g$  is the acceleration due to gravity,  $\theta_v$  is the virtual potential temperature and  $\overline{w' \theta_v'}$  (covariance of  $w$  and  $\theta_v$ ) is the virtual kinematic heat flux.  $u_*$  is defined as [Wyngaard, 2010],

$$u_* = \sqrt[4]{\overline{u' w'^2} + \overline{v' w'^2}}, \quad (1.12)$$

where  $\overline{u' w'}$  (covariance of  $u$  and  $w$ ) and  $\overline{v' w'}$  (covariance of  $v$  and  $w$ ) are the vertical fluxes of the horizontal momentum. The concept of  $\theta_v$  is important since atmosphere consists of moist and dry air, which influences the gas constants. For the ideal gas law to be valid one must resort to using the gas constant for moist air, which could be a bit cumbersome. Therefore the concept of virtual temperature is necessary, which denotes the temperature that the dry air must have in order to have the same pressure and density of the moist air. Having used virtual temperature the use of gas constant for dry air is then permitted. The term potential temperature denotes the temperature that the air parcel will have if it is adiabatically brought down to a standard reference pressure. It takes care of the variations in temperature due to altitude differences. Combining the concept of virtual temperature and potential temperature leads to virtual potential temperature. For mathematical details the reader is referred to [Stull, 1988].

## Chapter 2

# Introduction

Wind turbines have been and will be installed in different parts of the world where atmospheric conditions differ significantly from each other. Understanding and measuring atmospheric turbulence is vital to efficient harnessing of wind energy and to measuring the structural integrity of a wind turbine. Traditionally, meteorological mast (met-mast) anemometry has been used; in this method, either cup or sonic anemometers are mounted on slender booms at one or several heights to measure turbulence over a certain period of time. For wind energy purposes, much interest is focused on the turbulence of the wind and temperature, although some attention is also paid to other atmospheric variables such as pressure, humidity, density, etc.

Turbulence affects the wind turbines mainly in two ways: first, the fluctuations that are caused in the extracted wind power [Gottschall and Peinke, 2008, Kaiser et al., 2007], and second, the fluctuations in the loads on different components of a wind turbine [Sathe et al., 2013]. These fluctuations result in inefficient harnessing of wind energy and have the potential to inflict fatigue damage. Wind turbines are generally designed for a period of twenty years [Burton et al., 2001, IEC, 2005]. The size of a wind turbine has grown significantly over the past few decades. The upper tip of a modern wind turbine blade can easily reach heights up to 200 m above the ground. Thus measuring and understanding the turbulent wind field at higher heights is essential. It is very expensive to install and operate a met-mast at such heights for a sustained period of time. Especially offshore, the costs increase significantly owing to the large foundation needed to support the met-mast. Moreover, a met-mast cannot be moved from one place to another, thus limiting the physical range of the studies. Because of all these factors, measuring in the wake of a wind turbine (or multiple wakes) becomes quite a challenge. Lidars have the potential to counter these disadvantages of the met-mast anemometry. Recently, lidars have been used extensively for the measurement of the mean wind speed and wind profiling [Kindler et al., 2007, Peña et al., 2009, Smith et al., 2006, Wagner et al., 2011]. However, despite having been researched for years all over the world (particularly for meteorological studies), they have not yet been accepted for turbulence measurements. Lidars' lack of acceptance can be attributed to different reasons, such as large measurement volumes leading to spatial averaging of turbulence along the line-of-sight of its measurement axis, cross-contamination by different components of the wind field, low sampling rates, etc.

### 2.1 Basics of lidars

The introduction to this section is attributed to the contribution by Chris Slinger from ZephIR Ltd. There are many different types of lidars, which are capable of performing a diverse range

of tasks (e.g. 3D imaging and range finding, gas species detection, remote measurement of vibrations). In this report we restrict ourselves specifically with systems for the measurement of wind speed in the atmosphere. Such systems fall into two broad categories, namely the coherent lidars and direct detection lidars. Coherent lidar measures Doppler shifts by comparing the frequency of backscattered radiation to that of a reference beam via a light beating process, whereas direct detection lidar performs its frequency shift measurements by passing the light through an optical filter, such as a Fabry-Perot etalon. By operating in the ultra-violet, direct detection lidars can exploit molecular scattering processes, guaranteeing signal returns even in very clean air where there is an absence of scattering particles. Coherent wind lidar systems can be categorised according to their emission waveform (pulsed or continuous), waveband (visible, near-IR, far-IR), and their transmit/receive geometry (monostatic or bistatic). These notes concentrate specifically on continuous-wave (CW) and pulsed lidars.

### 2.1.1 Continuous Wave Lidar by Scott Wylie, ZephIR Ltd.

The basic principle of a CW lidar is to focus a continually transmitting laser beam at a particular measurement height (range) so that the Doppler shift of the backscattered light can be detected. If the motion of a particle along the beam direction is towards the lidar, it compresses the laser wavelength and increases its frequency (blue shift), while movement away from the lidar stretches the wavelength and reduces the frequency (red shift). This frequency shift can be measured by mixing the backscattered signal with a small portion of the original beam, allowing the difference in frequency to be detected. The resulting signal will oscillate at the so called beat frequency (the difference between the two signals being compared), which can be used to calculate the speed the particles are moving at (i.e. the wind speed).

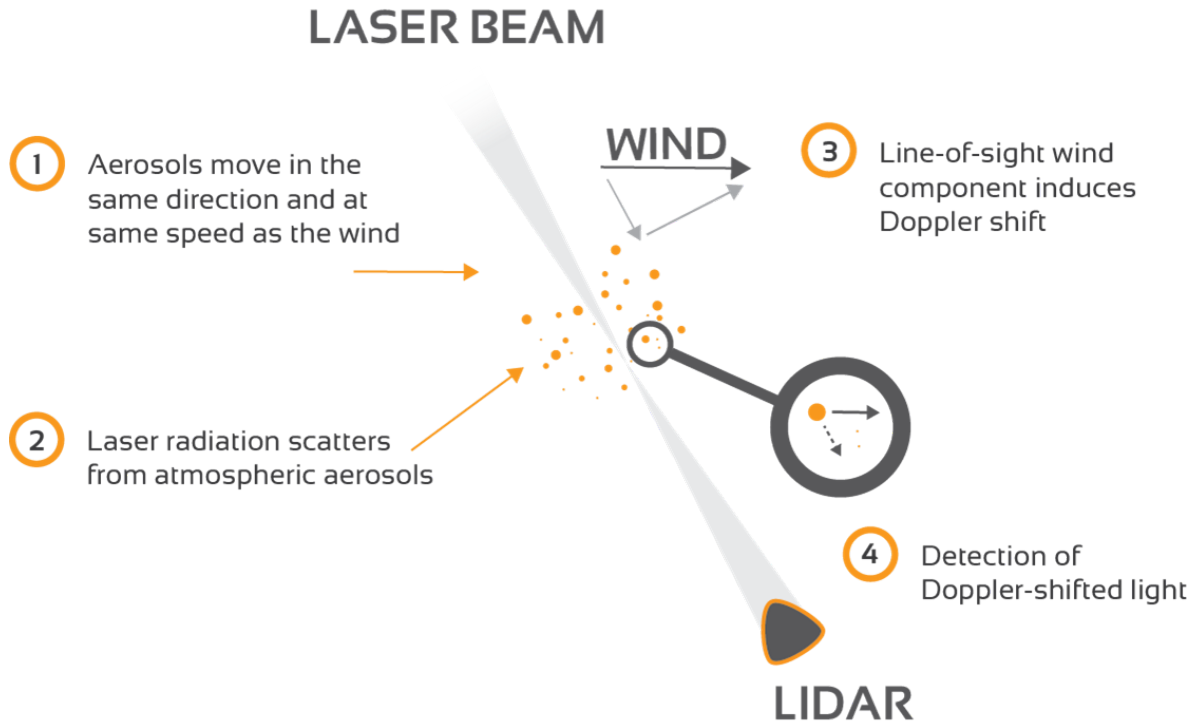


Figure 2.1: Principle of how a CW lidar detects backscatter from aerosols present in the atmosphere

If more than one measurement height is to be interrogated, the CW lidar will adjust its telescope to focus on each of the heights in sequence. The ranges possible from any CW

lidar are controlled by the focal properties of the systems optics. The focal depth of any telescope increases proportionally with the square of the distance to the measurement point of interest; the shorter the measurement distance, and the bigger the lens, the better defined the range and subsequent radial measurements will be confined to. This optical property limits the maximum range that can be achieved with a CW lidar; current CW technologies are producing reliable wind speed measurements at ranges up to approximately 200 meters. Figure 2.1 depicts the basic operation of how a CW lidar measures the wind speed. Unlike in a pulsed system, which utilises the time of flight to distinguish between measurements at different ranges, a CW lidar operates at a given range by focusing its beam. The distribution of wind speed with height is then achieved through wind profiling the process of continually scanning through each of the preset ranges in turn. A circular scan is typically used to provide a snapshot of the flow across a scan disk at each measurement range, with the rapid sampling rate inherent to CW lidar giving rise to measurements in the order of one second per range. Focusing of the beam results in a Lorentzian spatial weighting function along its axis, with a peak in the sensitivity located at the beam waist.

The minimum range that a CW lidar can measure is very short (in principle it is zero) whereas a pulsed system is effectively blind while the pulse is leaving the transmitter. This leads to a minimum range in the order of tens of meters for pulsed system, something in the region of 40-50 m is common. It is for eye safety reasons that a minimum measurement range of around 10 m is used for CW systems. A single lidar measurement will only provide the component of wind speed along its beam axis, and it is for this reason why a scan is needed to generate a measurement of the wind speed vector. A conical scan pattern is common practice here. As the beam moves, it intercepts the wind at different angles and builds up a series of measurements around its scan perimeter, which are then used to derive the wind speed vector. The peak Doppler shift is detected when the angle of the azimuth scan aligns with the upwind and downwind direction of the wind, with a Doppler shift close to zero arising when the azimuth angle is perpendicular to the flow. In uniform flow, a plot of the measured line-of-sight wind speed against the azimuth angle takes the form of a cosine wave, which is rectified in the case of a homodyne lidar system that cannot distinguish the sign of the Doppler shift.

### 2.1.1.1 Optics

Coupled with a transmitter and receiver (or transceiver in a homodyne system), the optics role is to provide a focused beam at a desired location. This location can be altered by changing the focus range or passing the beam through a scanning element such as a wedge (rotating prism). The angle of the wedge, if used in a wind profiling setup, is usually of the order 30° but can change based on the specific application e.g. a turbine mounted system may have a reduced wedge angle to account for its mounting position. If the CW lidar is a monostatic system then the backscattered light returns through the transmission optics, which can be isolated and then passed through the lidar for signal processing.

What is detected by the optics is the Doppler-shifted contribution generated by light scattering from any moving part of the atmosphere that is illuminated by the beam. The contribution from any point is weighted by the square of the beams intensity at that point. The sensitivity of the focused beam is at its peak at the beam waist, and tails off symmetrically either side. To a good approximation the axial weighting function for a CW monostatic lidar is given by a Lorentzian function [Sonnenschein and Horrigan, 1971],

$$\varphi(s) = \frac{1}{\pi} \frac{l}{l^2 + s^2}, \quad (2.1)$$

where  $\varphi(s)$  is the axial weighting function,  $s$  is the distance along the beam from the focus, and  $l$  is the Rayleigh length given as,

$$l = \frac{\lambda_b d_f^2}{\pi r_b^2}, \quad (2.2)$$

where  $\lambda_b$  is the wavelength of the emitted radiation, and  $r_b$  mm is the beam radius.

#### 2.1.1.2 Backscattering

The backscattered light detected by a CW lidar experiences a Doppler shift in frequency given by,

$$\delta f = 2 \frac{v_r}{\lambda_b} = 2 \frac{v_r f_b}{c}, \quad (2.3)$$

where  $f_b$  is the frequency of the emitted radiation,  $v_r$  is the radial velocity,  $\lambda_b$  is the wavelength of the emitted radiation, and  $c \approx 3 \times 10^8$  m/s is the speed of light. The backscattered signal detected by the lidar is made up of a range of different frequencies, which is a result of contributions from the different wind velocities (at strengths determined by the weighting function) measured over the probe length (space occupied by the focused lidar beam).

#### 2.1.1.3 Beat Phenomena

The detected Doppler-shifted radiation is optically mixed with a reference beam (sometimes called the local oscillator), which leads to the creation of the well-known beat phenomenon. Here the amplitude of the resulting signal oscillates at the difference frequency. For lidar, conveniently, this reduces the optical frequency of the Doppler shifted return from hundreds of GHz range to a signal more manageable in the MHz range. Detection of the beat signal is achieved by directing the optically-mixed beam onto a photodetector that measures fluctuations in the light's intensity. The photodetector outputs a measurable current (or voltage) that can be amplified for signal processing.

#### 2.1.1.4 Signal Processing

Figure 2.2 illustrates an example of signal processing for a CW lidar, but the details can vary from one lidar to the other. Spectral analysis is required to extract the relevant Doppler frequency information from the photodetector output, which for convenience is done digitally. The use of an analogue to digital converter (ADC) with a sampling rate of 100 MHz allows spectral analysis up to a maximum frequency of 50 MHz, corresponding to peak  $v_r \approx 38.8$  ms<sup>-1</sup> assuming use of a 30° wedge. Using digital Fourier transform (DFT), the spectra are analysed; a 512 point DFT gives rise to 256 points in the output spectrum with a bin width of  $\approx 200$  KHz, corresponding to  $v_r$  range of  $\approx 38.8$  ms<sup>-1</sup>. Each of the line of sight measurements are sampled, representing  $\approx 5$   $\mu$ s of data; successive DFTs are then calculated, and the resulting voltage spectra are squared in order to generate a power spectrum. These power spectra are then averaged to find a mean spectrum for the averaging period. The random noise contained in the signal reduces with the square root of the number of averages taken, with the sensitivity increasing by the same factor. 4000 averages are taken for each line of sight measurement, which gives a data rate close to 50 Hz and a measurement time of around 20 ms. The width of the Doppler spectrum is determined by the following:

- Instrumental width – This is closely linked to the DFT bin width mentioned earlier



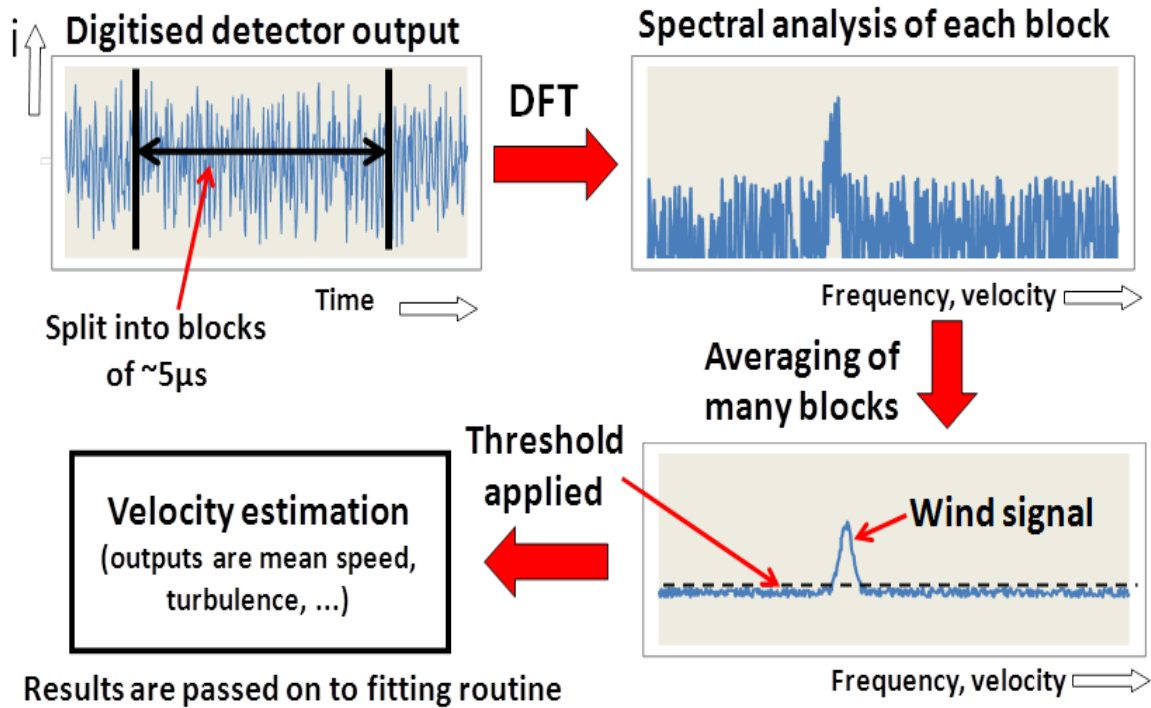


Figure 2.2: Typical signal processing stages undertaken to produce wind vector from line of sight measurements

- **Transit-time broadening** – This is associated with the scan (assuming it is conical), the beam passes through the aerosols in a timescale of  $\approx 10 - 15 \mu\text{s}$ , corresponding to a broadening of the order 200 KHz
- **Turbulence broadening** – This is an effect from measuring over a volume, as opposed to point measurement such as a cup. When a large volume is probed in the atmosphere, a range of Doppler shifts can be detected, corresponding to parts of the atmosphere moving at different speeds. The impact of this is to have more than one peak in the detected spectrum. In general, this contribution will be increased during times of high turbulence and shear, meaning there is a potential to use this as an indication or measure of turbulence at a site.

### 2.1.2 Pulsed Lidar

The description of the operating principles of a pulsed lidar is taken from [Vasiljevic \[2014\]](#). As described in chapter 1, the notations used in this section may differ slightly than the ones used in the rest of the report. However, this does not hinder the readability of the rest of the report, as consistent notations are used throughout. A coherent pulsed Doppler lidar performs three fundamental processes that enable measurements of the radial velocity:

1. Emission of laser pulses
2. Acquisition of the backscattered light
3. Analysis of the acquired backscattered light

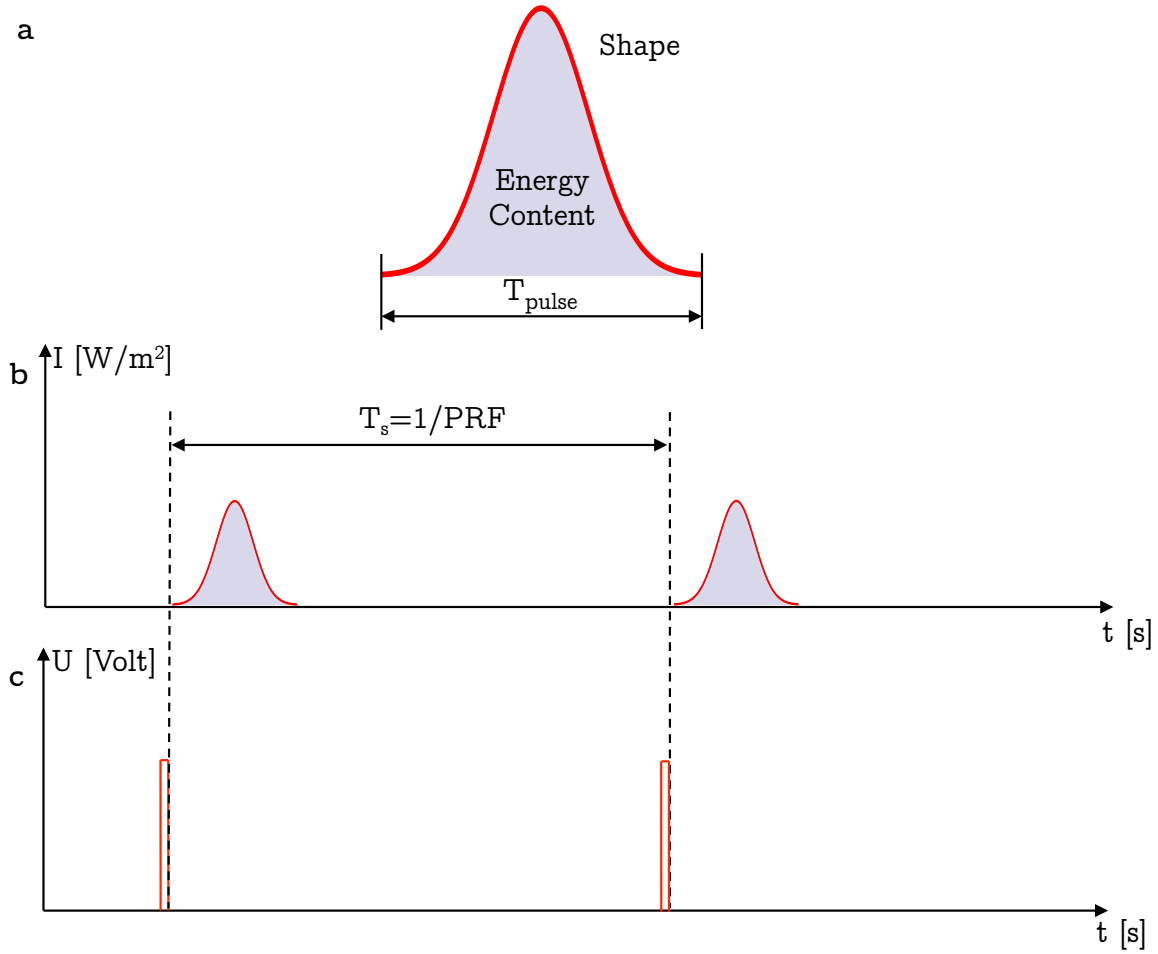


Figure 2.3: Emission: a - laser pulse, b - pulse train, c - trigger signal

Table 2.1: Type of laser pulses

Type	Wavelength (nm)	Temporal Length (ns)	Energy ( $\mu\text{J}$ )	PRF (kHz)
Long	1543	400	100	10
Middle	1543	200	50	20

#### 2.1.2.1 Emission of the laser light

A measurement process starts with the emission of the laser pulses. Each emitted laser pulse has a characteristic Gaussian shape with a certain temporal length  $T_{\text{pulse}}$ , energy content  $E$  and wavelength  $\lambda_b$  (see Fig. 2.3a). Laser pulses are usually emitted in bursts that last continuously over some period of time (see Fig. 2.3b). The emission frequency is constant and is known as the pulse repetition frequency (PRF). Different type of laser pulses can be emitted, where two examples are listed in table 2.1. The Long pulses contain more energy than the Middle pulses, and due to the two times larger temporal length the aerosols particles at any distance are exposed to the laser light for a longer period. This results in higher carrier-to-noise ratio (CNR), which directly influences the maximum distance from which the radial velocity can be retrieved. The drawback of the Long pulses is that the retrieved radial velocity is characterized by the two times larger range resolution than in the case of the Middle pulses, which means that eddies smaller than the range resolution are filtered out. Typically

the Long pulses are used to retrieve the radial velocity from distances of up to 8 kilometers. On the other hand, the Middle pulses are suitable for the retrieval of radial velocity from distances of up to 4 kilometers with the half range resolution of the Long pulse.

The emission process begins with the start of the trigger signal (see Fig. 2.3c). Each time the pulse generator receives a trigger, it sends an analog signal of the pulse shape and a copy of the trigger to the acousto-optic modulator (AOM). Based on these two input signals and the low-energy laser light from the CW laser, the AOM forms a low-energy laser pulse. In comparison to the original CW light, the laser pulse frequency is shifted to  $f_b = f_{CW} + f_{AOM}$ , where  $f_{CW}$  is the frequency of the monochromatic low-energy laser light, and  $f_{AOM}$  is the frequency of the AOM. The AOM frequency is equal to about 60 MHz, and the shift in the frequency allows determining the retrieved radial velocity sign.

Once the low-energy laser pulse is formed, it is directed to the Erbium-doped fiber amplifiers (EDFA), which increases the energy content of the pulse. This forms the high-energy laser pulse. After the EDFA, the high-energy laser pulse passes through the optical circulator and telescope. The optical circulator has the role to separate directions of the outgoing laser pulses and the incoming backscattered light. By using the optical circulator, the transmitter of the laser pulses and the receiver of the backscattered light can both use the same optical path. The telescope is used to magnify the laser beam and to focus the beam at a certain distance. The magnification reduces the beam divergence in the far field, while the focusing is used to optimize the distribution of the laser beam power along the distance.

### 2.1.2.2 Acquisition of the backscattered light

As the laser pulse propagates through the atmosphere, along a direction given by the azimuth and elevation angles of the scanner head, it interacts with dispersed moving aerosol particles in the atmosphere. It is assumed that the particle velocities are equal to the wind velocity. Due to the optical Doppler effect, the particles perceive the incoming laser pulse light with slightly shifted frequency  $f_d$  (also called as the Doppler frequency), where the difference in frequency corresponds to the velocity of the particles projected on the laser pulse propagation path, i.e. radial or LOS velocity. In the interaction between the particles and laser pulse, a small portion of the laser pulse light is reflected from the moving particles back to the lidar. Because of the movement of the particles, the backscattered light has the original frequency  $f_b$  shifted by twice the radial velocity divided by the wavelength of the emitted radiation. This shift in the frequency of the backscattered light is commonly known as the Doppler shift given as,

$$\delta f = 2 \frac{v_r}{\lambda_b}, \quad (2.4)$$

where  $v_r$  is the radial velocity. The sign of the Doppler shift could be positive or negative for the particles moving away from the lidar depending on the conventions used in a particular type of lidar. Due to the laser pulse's propagation through the atmosphere, the lidar continuously receives the backscattered light from different distances and thus the information about the radial velocity. Using the range gating technique, distinction between distances is achieved by using the backscattered lights time of arrival in relation to the start of the laser pulse.

Once the backscattered light reaches the lidar, it follows the path of the outgoing laser pulses. It reflects on the mirrors, and it passes through the telescope after which it enters the optical circulator. Through the system of optical fibers, the backscattered light is directed towards the optical mixer, where it is optically mixed with the copy of the low-energy CW laser light, known as the local oscillator (LO) beam. The mixing of two light signals leads to the 'beat' phenomenon, in which the amplitude of the resulting light oscillates at the frequency

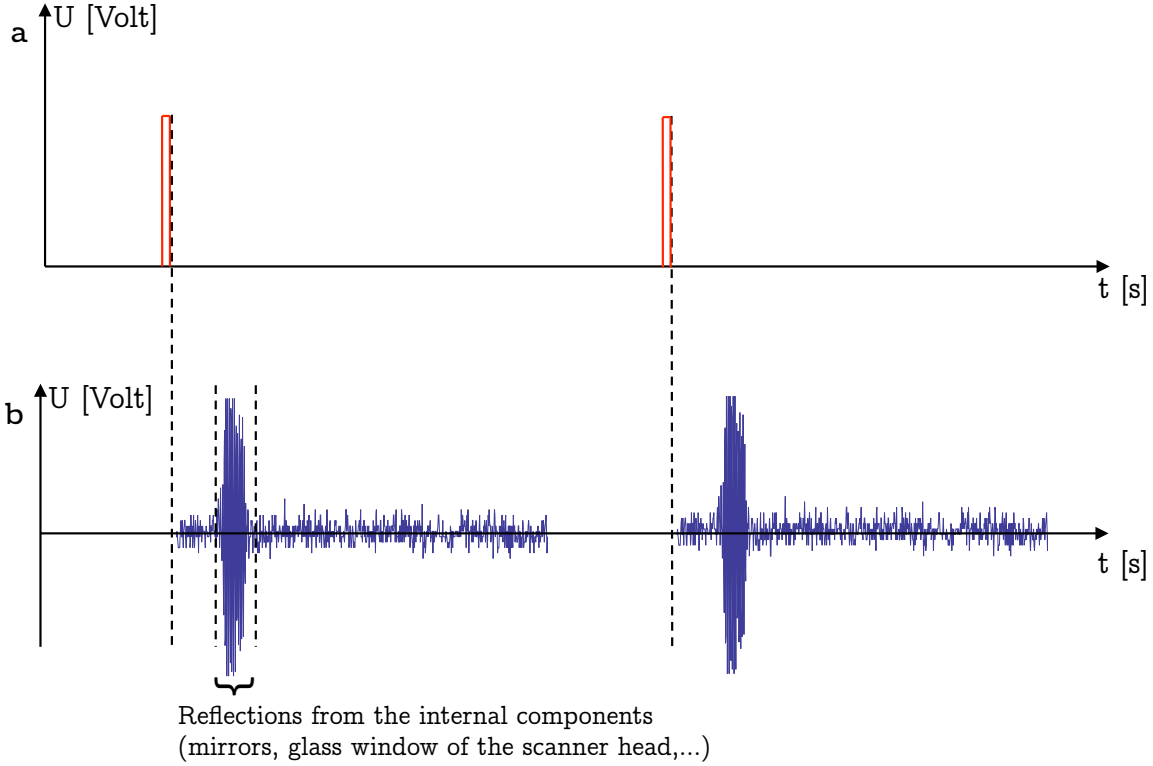


Figure 2.4: Emission: a - trigger signal, b - acquired analog signal

difference between two light signals. This beating light signal is focused on the photodetector that transforms the light signal into an analog signal that follows the oscillation of the light intensity. The acquisition of the photodetector output occurs each time the acquisition board receives a trigger from the motion controller (see Fig. 2.4). The number of sample points of the digitized signal determines the maximum distance at which the radial velocity will be retrieved.

### 2.1.2.3 Analysis of the acquired backscattered light

The radial velocity at a distance  $d$  (note that in the rest of the report  $d_f$  is used instead) can be retrieved from the return of a single laser pulse by the estimation of the mean Doppler shift  $\delta f$  from  $M$  sample points of the corresponding digitized output of the photodetector. These  $M$  sample points define the observation time  $T_{\text{FFT}} = MT_s$ , and they include the information regarding the backscattered light that originates from a range of distances  $(d - \Delta d/2, d + \Delta d/2)$  centered at the distance  $d$  (see Fig. 2.5). If the finite discrete signal, given with  $M$  sample points, is transformed to the frequency domain, and spectrum of the transformed signal calculated, then by applying a frequency estimator on the spectrum, e.g. a Maximum Likelihood Estimator (MLE) [Valla, 2005], the frequency of the spectral peak can be estimated. Subtracting  $f_{\text{AOM}}$  from the estimated frequency yields the mean Doppler shift of the backscattered light from the range of distance centered at the distance  $d$ . Along with the Doppler shift, the MLE estimates the spectral broadening and CNR from the signal spectrum.

In order to express the signal of  $M$  sample points in terms of the spectrum, the observation time  $T_{\text{FFT}}$  should be larger than the backscattered light correlation time, which can be approximated as the temporal length of the emitted laser pulse  $T_{\text{pulse}}$  [Frehlich et al., 1994]. The narrower the spectrum is, the more sample points are used to derive the spectrum. This

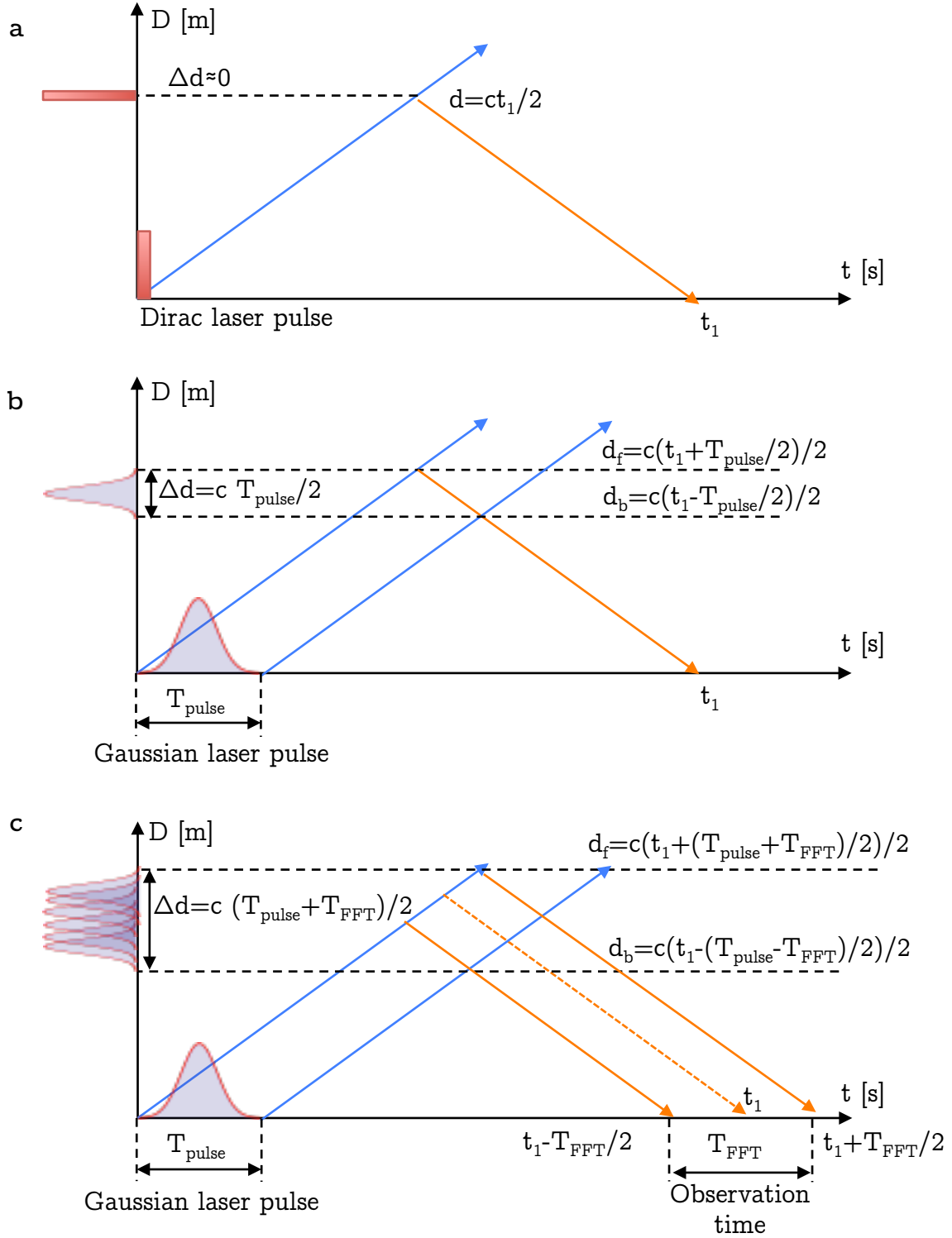


Figure 2.5: Retrieval of the radial velocity: a - one sample point of the dirac return, b - one sample point of the Gaussian return, c - M sample points of the Gaussian return

results in the improved velocity resolution, since each frequency bin in the spectrum will be defined on the smaller frequency range. The consequence of this is an increase in the range resolution, since more sample points mean bigger range of distance from which the

backscattered light is acquired and analyzed. Due to the tradeoff between the velocity and range resolution, the observation time  $T_{\text{FFT}}$  is usually set to the temporal length of the emitted laser pulse  $T_{\text{pulse}}$ , which provides one independent retrieval of the radial velocity per observation time.

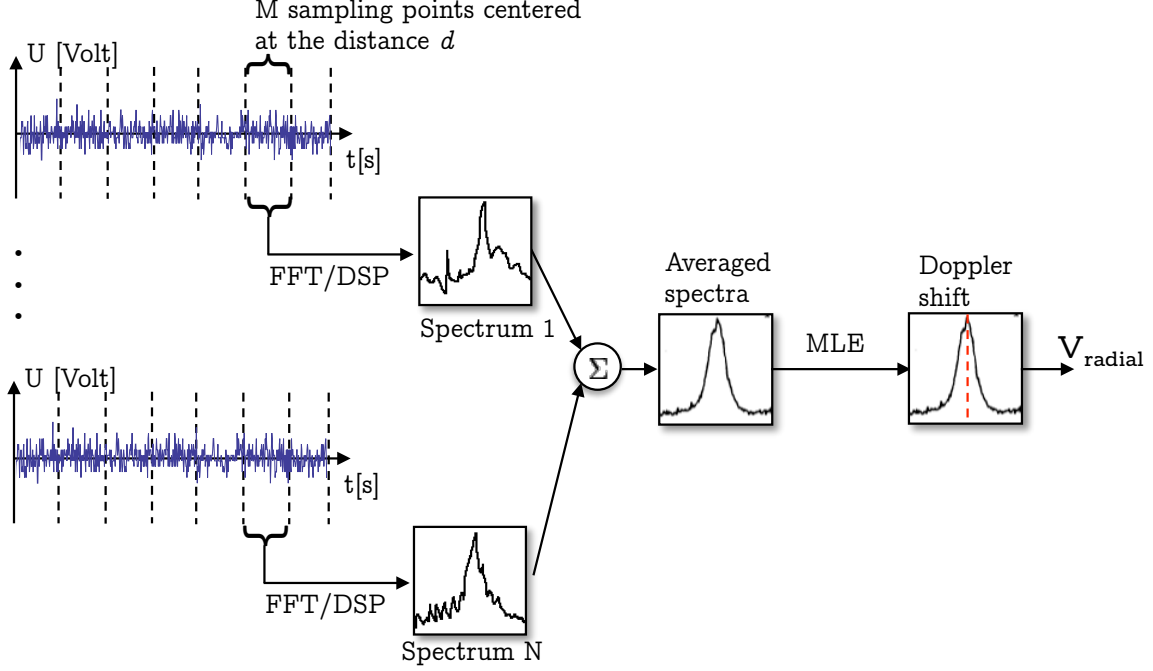


Figure 2.6: Accumulation Method

The retrieval of the radial velocity from a single laser pulse return encompasses the random error that originates from the uncorrelated noise [Frehlich, 2001], which leads to the incorrect estimate of the spectral peak. As an alternative, the estimation of the mean Doppler shift from  $N$  accumulations of the laser pulse returns leads to the suppression of the random error and improvement of the Doppler shift estimation accuracy [Davies and Collier, 1999]. In this method, the frequency estimator is applied on the averaged sum of  $N$  spectra (see Fig. 2.6). It has been shown in Frehlich et al. [1994] that the number of accumulations  $N$  of the order of 10 is useful for eliminating the incorrect estimates of the radial velocity at low CNR.

## 2.2 Turbulence statistics relevant for wind energy

According to IEC [2005] standards, a wind turbine should be designed for different classes of turbulence intensities. The turbulence intensity is defined according to Eq. (1.3). It is thus crucial to perform measurements of  $\langle u'^2 \rangle$ . Apart from  $I$ , it is also important to measure the mean wind speed profile, which is dependent on the velocity covariances  $\langle u'w' \rangle$  and  $\langle v'w' \rangle$  [Wyngaard, 2010]. The diagonal components of  $\mathbf{R}$ , i.e.,  $\langle u'^2 \rangle$ ,  $\langle v'^2 \rangle$  and  $\langle w'^2 \rangle$  influence the loads significantly. Thus for wind energy purposes, it is very important to measure  $R_{ij}$ .

A current practice in the wind energy industry to perform load simulations is that a turbulent wind field is generated using either the Mann [1994] model or an empirical Kaimal et al. [1972] spectrum is combined with some coherence model [IEC, 2005]. As discussed in chapter 1, the need to measure  $\varepsilon$  and  $\mathcal{L}$  is then clearly evident. These parameters are normally obtained by fitting the Mann [1994] model to the measurements of  $F_{ij}(k_1)$ , which could be obtained using lidars.  $\mathcal{L}$  and  $\text{coh}_{ij}$  are important for estimating the loads and

wake meandering [Larsen et al., 2008]. The influence of atmospheric stability on wind speed profile and on wind turbine loads is becoming increasingly evident [Sathe et al., 2011a, 2013]. For this reason, measurement of  $\langle w'\theta' \rangle$  is quite important for wind energy. According to Lenschow et al. [1994],  $\ell_{ij}$  is useful in estimating the averaging time required to keep the random errors below a certain threshold for a particular turbulence statistic, and hence is a desirable measurement quantity for wind energy purposes.

Recently, lidars are being contemplated to be used for wind turbine control. The concept is such that the lidar is either placed on a nacelle of a wind turbine [Schlipf et al., 2013], or mounted inside a spinner [Mikkelsen et al., 2013, Simley et al., 2013] in order to detect the incoming wind field and carry out a feed-forward control to reduce the structural loads on a wind turbine. The degree to which such a concept can be successfully applied depends on how well the lidars are able to detect the incoming turbulent structures. From Sathe et al. [2013] we understand that different components of a wind turbine are affected by different scales of turbulent structures. It is thus important to be able to detect the range of turbulence scales, up to the order of or less than the probe volume length.

## Chapter 3

# Measurement Configurations

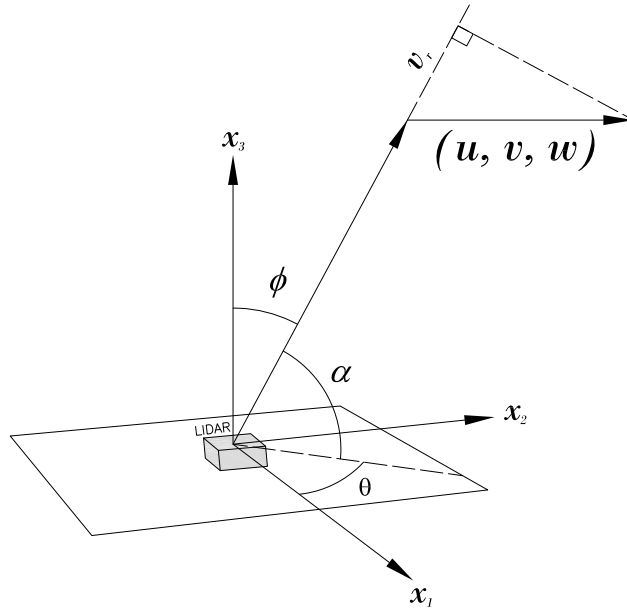


Figure 3.1: Coordinate system of a lidar

In order to understand different measurement configurations better, we first define a coordinate system in which a lidar performs measurements. It is to be noted that the coordinate system defined in this section is not a universally accepted system, but simply a reference based on which different measurement configurations could be understood. We choose a base coordinate system in accordance with that defined in chapter 1. As shown in Fig. 3.1, at a given instant of time if we assume that a lidar measures at a point, and that the lidar beam is inclined at a certain zenith angle  $\phi$  (in some literature the complement of  $\phi$  is used, which is called as the elevation angle  $\alpha = 90^\circ - \phi$ ) from the vertical axis, and makes an azimuth angle  $\theta$  with respect to the axes in the horizontal plane, then the radial velocity (also called as the line-of-sight velocity) can be mathematically written as,

$$v_r(\phi, \theta, d_f) = \mathbf{n}(\phi, \theta) \cdot \mathbf{v}(\mathbf{n}(\phi, \theta)d_f), \quad (3.1)$$

where  $v_r$  is the radial velocity measured at a point,  $\mathbf{n} = (\cos \theta \sin \phi, \sin \theta \sin \phi, \cos \phi)$  is the unit directional vector for a given  $\phi$  and  $\theta$ ,  $d_f$  is the distance at which the measurement is obtained, and  $\mathbf{v} = (u, v, w)$  is the wind vector. For simplicity in the rest of the report, it is assumed that the streamline coordinate system is aligned with the coordinate system of the



lidar such that  $u$  is along the positive  $x_1$  axis. In Eq. (3.1), we have implicitly assumed that  $v_r$  is positive for the wind going away from the lidar axis, the coordinate system is right-handed, and  $u$  is aligned with the  $x_1$  axis in a horizontal plane, i.e. from west to east. In reality, a lidar never receives backscatter from exactly a point, but from all over the physical space. Fortunately the transverse dimensions of a lidar beam are much smaller than the longitudinal dimensional, and for all practical purposes we can consider that the backscatter is received only along the lidar beam axis. Mathematically the radial velocity can be represented as the convolved signal,

$$\tilde{v}_r(\phi, \theta, d_f) = \int_{-\infty}^{\infty} \varphi(s) \mathbf{n}(\phi, \theta) \cdot \mathbf{v}(\mathbf{n}(\phi, \theta)(d_f + s)) ds, \quad (3.2)$$

where  $\tilde{v}_r$  is the weighted average radial velocity,  $\varphi(s)$  is any weighting function integrating to one that depends on the type of lidar, i.e. a continuous wave (c-w) lidar or a pulsed lidar, and  $s$  is the distance along the beam from the measurement point of interest. In the following sections, where possible, only the point representation of the radial velocity, i.e.  $v_r$  will be used for simplicity.

### 3.1 Commercial configurations

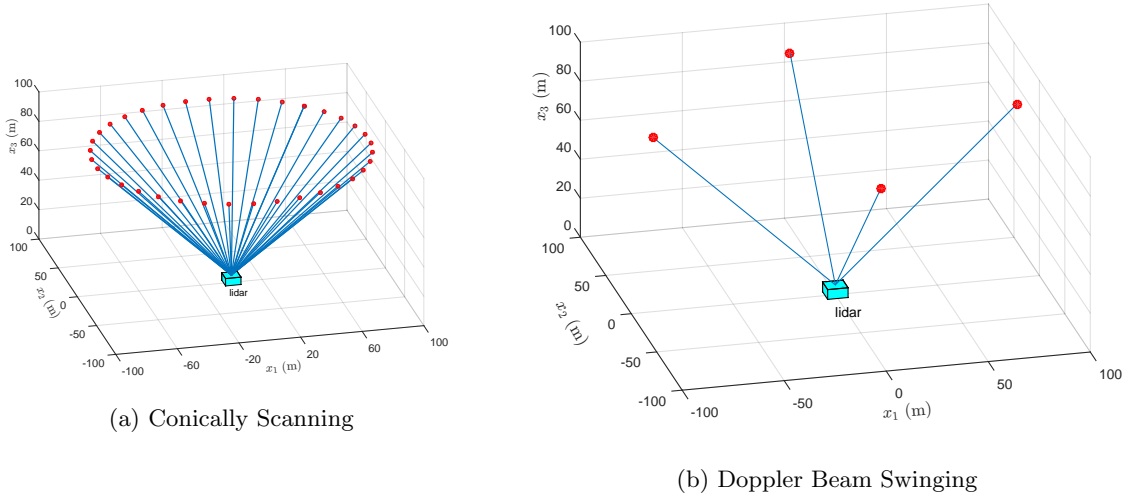


Figure 3.2: The two most commonly used measurement configurations in commercial lidars

Figure 3.2 shows the two most commonly used scanning configurations by commercial lidars. The conically scanning configuration performs a conical scan, where several measurements of the radial velocity ( $v_r$ ) are performed over the base of a cone. The Doppler Beam Swinging (DBS) scanning configuration also performs a conical scan, but with  $v_r$  measurements of only a few beams on the base of the cone. Here we use four beams as an example, but it could also be three beams. Both configurations use the so-called velocity azimuth display (VAD) method of data processing to deduce the wind field components  $u$ ,  $v$  and  $w$ . In principle to deduce the wind field components we only need three  $v_r$  measurements at different  $\theta$  and  $\phi$ . However for the conically scanning configuration (Fig. 3.2a), we have several measurements of  $v_r$ , which results in an over-determined system. Least-squares analysis is

thus used to deduce the wind field components as,

$$\begin{aligned} u &= \frac{1}{\pi \sin \phi} \int_0^{2\pi} v_r \cos \theta \, d\theta, \\ v &= \frac{1}{\pi \sin \phi} \int_0^{2\pi} v_r \sin \theta \, d\theta \\ w &= \frac{1}{2\pi \cos \phi} \int_0^{2\pi} v_r \, d\theta, \end{aligned} \quad (3.3)$$

where the argument of  $v_r$  is dropped for simplicity. For the DBS scanning configuration (Fig. 3.2b), let us denote the beams in the positive and negative  $x_1$  direction as east (E) and west (W) respectively. Similarly the beams in the positive and negative  $x_2$  direction are defined as north (N) and south (S) respectively. The wind field components are then deduced as,

$$\begin{aligned} u &= \frac{v_{rE} - v_{rW}}{2 \sin \phi}, \\ v &= \frac{v_{rN} - v_{rS}}{2 \sin \phi}, \\ w &= \frac{v_{rE} + v_{rN} + v_{rW} + v_{rS}}{4 \cos \phi}. \end{aligned} \quad (3.4)$$

Equations (3.3) or (3.4) denote the VAD method of data processing that involves estimating the wind field components by combining  $v_r$  measurements from several beams for each scan. For a given averaging period the deduced wind field components from each scan produce a time series, which are used to estimate the turbulence statistics.

### 3.1.1 Estimating components of $\mathbf{R}$ using VAD technique of processing lidar data

In wind energy, we are usually interested in statistics defined by Eqs. (1.7) or (1.9) (see also section 2.2). However, different lidar data processing techniques produce different estimates of turbulence statistics than those given by Eqs. (1.7) or (1.9). Commercial lidars usually use the VAD/DBS technique of data processing, where the  $v_r$  measurements at different azimuth angles are combined to deduce  $u$ ,  $v$  and  $w$ . The deduced time series of the wind field components is further processed to estimate turbulence statistics within a given averaging period. As a result, we do not obtain the standard turbulence statistics defined by Eq. (1.7), but some filtered (on small scales) statistic and contaminated by cross-correlations between different wind field components. Mathematically, it is given as,

$$R_{mn}^{\text{lidar}} = \int \Phi_{ij}(\mathbf{k}) X_i^m(\mathbf{k}) X_j^{*n}(\mathbf{k}) \, d\mathbf{k}, \quad (3.5)$$

Where the subscript lidar denotes the estimated statistic using the lidar measurements,  $\mathbf{X}(\mathbf{k})$  is the filter function in Fourier domain, which depends on the type of lidar (CW or pulsed) and the Reynolds stress tensor component of interest.

The detailed derivation of Eq. (3.5) can be found in Sathe et al. [2011b], but even without going through the mathematical details, if we simply compare Eqs. (3.5) and (1.7), it is clear that they are very different from each other. The function  $\mathbf{X}(\mathbf{k})$  acts as a filter to smaller scales of turbulence, whereas its combination with the spectral velocity tensor  $\Phi(\mathbf{k})$  (by applying the Einstein summation notation) denotes the contamination due to the cross-correlation of different wind field components. These two effects tend to counter each other, and therefore sometimes turbulence estimates from the VAD method can have comparable accuracy with

those estimated from the reference instruments. In colloquial language it could be understood as 'getting it right for the wrong reasons'. Hence one should be careful in using the VAD method to estimate turbulence statistics, since the accuracy of turbulence estimates may not be reproducible.

### 3.1.2 Estimating $\varepsilon$ using conically scanning lidar

In order to use this method the scanning speed of the lidar must be much larger than the advection speed of turbulence. An expression can then be derived for the radial velocity structure function for different separation distances  $d_f\delta$ , on the base of the scanning cone, where  $\delta = 2\sin^{-1}(\sin\phi\sin\theta)$  is the angle subtended by the two lidar beams in a conical scan. Banakh et al. [1996] were the first to formulate mathematical expressions, but Kristensen et al. [2012] re-derived their original expressions, where an additional  $R(0)$  term was added. The contribution due to random instrumental noise was however neglected that was considered in Banakh et al. [1996]. For modern lidar systems, the instrumental noise can be neglected [Mann et al., 2009].

Two approaches were chosen in the derivation by Kristensen et al. [2012]: time-domain autocorrelation approach, and the Fourier-domain wave-number approach. The Fourier-domain approach is derived for a CW lidar (assuming a Lorentzian function), whereas the time domain approach provides expressions as a function of  $\varphi(s)$ . By using appropriate  $\varphi(s)$ , the time-domain expressions can be applied to a CW or a pulsed lidar. The equations using both approaches are as follows. In the time domain,

$$\begin{aligned}\tilde{D}(\delta) = & 2(1 - \cos\delta)R(0) + \frac{9}{55}\Gamma\left(\frac{1}{3}\right)C(\varepsilon d_f)^{2/3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varphi(s'_1)\varphi(s'_2) \\ & \cdot \left( 3\left( (s'_2 - s'_1)^2 + 4s'_1s'_2\sin^2(\delta/2) \right)^{1/3} \cos\delta - |s'_2 - s'_1|^{2/3} \right) \\ & + \frac{s'_1s'_2\sin^2\delta}{((s'_2 - s'_1)^2 + 4s'_1s'_2\sin^2(\delta/2))^{2/3}} \Big) ds'_1 ds'_2, \quad (3.6)\end{aligned}$$

where  $\tilde{D}(\delta)$  is the filtered radial velocity structure function for a separation distance  $d_f\delta$ , on the base of the cone,  $R(0) = \langle u'^2 \rangle = \langle v'^2 \rangle = \langle w'^2 \rangle$  for isotropic turbulence, and  $s'_1 = s_1/d_f$ ,  $s'_2 = s_2/d_f$  are non-dimensional variables. In the Fourier domain, for a C-W lidar,

$$\begin{aligned}\tilde{D}(\delta) = & 2(1 - \cos\delta)R(0) + C(\varepsilon d_f)^{2/3} \frac{3}{55}\Gamma\left(\frac{1}{3}\right) \\ & \left( \frac{3}{\sqrt[3]{2}}(1 + 7\cos\delta)\sin^{2/3}(\delta/2) - 18\left(\frac{d_f}{l}\right)^{-2/3} \right. \\ & + \frac{1}{\pi}\left(\frac{2d_f}{l}\right)^{-2/3} \int_0^{\pi/2} \frac{\Gamma(1/2)\Gamma(1/3)}{\Gamma(5/6)}(7\cos\delta - 4\cos(2\xi)) \\ & \cdot \left( 2\cos\left(\frac{2}{3}\tan^{-1}\left(\frac{4d_f\sin(\delta/2)\sin\xi}{l(|\cos(\xi + \delta/2)| + |\cos(\xi - \delta/2)|)}\right)\right) \right. \\ & \cdot \left( (|\cos(\xi + \delta/2)| + |\cos(\xi - \delta/2)|)^2 + 16\left(\frac{d_f}{l}\right)^2 \sin^2(\delta/2)\sin^2\xi \right)^{1/3} \\ & \left. \left. - \left(4\frac{d_f}{l}\sin(\delta/2)\sin\xi\right)^{2/3} \right) d\xi \right). \quad (3.7)\end{aligned}$$

The key to using this method is to appropriately select  $d_f\delta \ll \mathcal{L}$ , so that turbulence is measured in the inertial subrange, and is locally isotropic.  $\tilde{D}(\delta)$  can be measured using

a lidar; then, by knowing  $R(0)$ , we can estimate  $\varepsilon$ . [Banakh et al. \[1996\]](#) did not include the  $R(0)$  term in their equation, perhaps because at  $\delta \ll \pi/2$ , and  $d_f \gg \mathcal{L}$ , this term is negligible. The advantage of using Eq. (3.7) is that we need to solve only a single integral numerically, whereas in Eq. (3.6) we need to solve a double integral numerically, and that may increase the numerical error. The estimation of  $R(0)$  can be quite challenging, since it also contains information about the large-scale turbulence. [Kristensen et al. \[2012\]](#) used empirical models for convective turbulence [[Kristensen et al., 1989](#)] and estimated that  $R(0) = 1.74 \varepsilon^{2/3} (d_f \cos \phi)^{2/3}$ . Alternatively, one may use the [von Kármán \[1948\]](#) energy spectrum and derive expressions for  $R(0)$ .

## 3.2 Research configurations

### 3.2.1 Staring mode

This is the simplest of all the measurement configurations, where a lidar beam constantly points only at one height with a given  $\theta$  and  $\phi$ . Fig. 3.1 illustrates this measurement configuration. Because  $v_r$  is a function of three wind field components, a single beam cannot be used to retrieve  $u$ ,  $v$  and  $w$ . Despite its simplicity, one could use this configuration to estimate a small scale turbulence parameter, namely the turbulent kinetic energy dissipation rate  $\varepsilon$  [[Banakh and Smalikho, 1997b](#), [Frehlich et al., 1998](#), [Smalikho, 1995](#)]. Two approaches can be used,

- Doppler spectrum width
- Radial velocity structure function

#### 3.2.1.1 Estimating $\varepsilon$ using Doppler spectrum width

This method requires access to the raw Doppler spectra data, which is used to estimate  $v_r$ . Only the mathematical formulation for a CW lidar is provided, since the mathematics for a pulsed lidar is extremely complicated, and interested readers can refer to [Smalikho et al. \[2005\]](#). For a CW lidar, if we define  $l = \lambda_b d_f^2 / \pi r_b^2$  as the Rayleigh length corresponding to the filtering of the small scale turbulence, where  $\lambda_b$  is the wavelength of the emitted radiation,  $r_b$  is the beam radius, and  $d_f$  is the distance at which the measurements are obtained, then

$$\langle \sigma_s^2 \rangle = 1.22 C \varepsilon^{2/3} l^{2/3}, \quad (3.8)$$

where  $\langle \sigma_s^2 \rangle$  is the second central moment of the Doppler spectrum (or its width), and  $C \approx 1.5$  is the universal Kolmogorov constant. For detailed derivation of Eq. (3.8) the readers are referred to [Smalikho \[1995\]](#).

$\langle \sigma_s^2 \rangle$  can be measured and  $l$  is known, so  $\varepsilon$  can be estimated. The limitation of this method is that Eq. (3.8) can only be used when  $l \ll \mathcal{L}$ , where  $\mathcal{L}$  is the outer scale of turbulence. Moreover, the effect of mean radial velocity gradient within the probe volume has not been taken into account. Equation (3.8) states that if there is no turbulence, then the Doppler spectral width should be zero. However, if there is a mean change of  $v_r$  with  $s$  (within the probe volume) then there is an additional term proportional to  $l^2$ . If the lidar is C-W and the shear is linear, then the coefficient of  $l^2$  is infinite [[Mann et al., 2010](#)] and we cannot use this method.

#### 3.2.1.2 Estimating $\varepsilon$ using radial velocity structure function

The main challenge in using this method lies in appropriately selecting an inertial subrange from the lidar data, where an assumption of isotropy of the turbulence can reasonably be

assumed to be true [Pope, 2000]. A consequence of this assumption is that for a point measurement, the turbulence spectrum (or equivalently the structure function) becomes proportional to  $\varepsilon^{2/3}$  only. As an example, the structure function of the  $u$  component in the inertial subrange can be expressed as [Pope, 2000]:

$$D_{11}(r_1) = C_2 \varepsilon^{2/3} r_1^{2/3}, \quad (3.9)$$

where  $D_{11}(r_1)$  is the one-dimensional structure function of the longitudinal wind field component  $u$ , and  $C_2 \approx 2$  is the Kolmogorov constant related to  $D_{11}(r_1)$ . For a lidar, owing to presence of a large measurement volume we need to consider the weighting function  $\varphi(s)$  within the measurement volume, and as a result the equation of the structure function becomes much more complicated. Moreover  $\varphi(s)$  is different for different types of lidars, i.e. a CW or a pulsed lidar.

### 3.2.1.2.1 Continuous Wave lidar

The weighting function  $\varphi(s)$  for a CW lidar can reasonably be assumed to be Lorentzian [Sonnenschein and Horrigan, 1971], which results in filtering of the small scale turbulence. For a staring lidar one could only estimate the radial velocity structure function, which mathematically can be expressed as [Kristensen et al., 2011, Smalikho, 1995],

$$\tilde{D}(r_1) = C \varepsilon^{2/3} l^{2/3} \frac{\Gamma(1/3)}{5\sqrt{\pi}\Gamma(5/6)} \int_0^{2\pi} \left(1 - \frac{8}{11} \cos^2 \xi\right) \Psi(r_1, \beta, \xi) d\xi, \quad (3.10)$$

where  $\tilde{D}(r_1)$  is the filtered radial velocity structure function measured by the lidar,  $r_1 = \langle u \rangle t$  is the separation distance along the  $x_1$  axis,  $\Gamma(n) = \int_0^\infty x^{n-1} \exp(-x) dx$  is the gamma function,  $\beta = \arcsin(\sqrt{(d_f \cos \phi)^2 + (d_f \sin \phi \sin \Theta)^2}/d_f)$  is the angle between the lidar beam and the mean wind  $\langle u \rangle$ , and

$$\begin{aligned} \Psi(r_1, \beta, \xi) = & \frac{3}{2} \Gamma\left(\frac{1}{3}\right) \left( \left( \cos^2 \xi + \left(\frac{r_1}{l}\right)^2 \cos^2(\xi + \beta) \right)^{1/3} \right. \\ & \left. \cdot \cos\left(\frac{2}{3} \tan^{-1}\left(\frac{r_1}{l} \left| \frac{\cos(\xi + \beta)}{\cos \xi} \right| \right)\right) - |\cos \xi|^{2/3} \right). \end{aligned} \quad (3.11)$$

$r_1$  is computed using the Taylor's hypothesis [Taylor, 1938], where turbulence is assumed to be advected by the mean wind  $\langle u \rangle$  in time  $t$ .

For the measured and known parameters  $\tilde{D}(r_1)$ ,  $\beta$ ,  $r_1$ ,  $l$  and  $C$ , the unknown  $\varepsilon$  can be estimated, where the one-dimensional integral in Eq. (3.10) can be solved numerically. It is to be noted that due to the dependence of  $\tilde{D}(r_1)$  on  $\beta$ , it is essential to estimate  $\Theta$ , which could be done by alternating between staring and conically/DBS scanning, or by using estimated  $\Theta$  from the met-mast anemometry. The advantage of alternating the scans is then obvious, since it obviates the need for a met-mast. Comparing Eqs. (3.9) and (3.10), it is clear that estimate of  $\varepsilon$  using a lidar is much more complicated, but nevertheless possible.

### 3.2.1.2.2 Pulsed lidar

For a pulsed lidar, different shapes of weighting functions have been suggested, e.g. triangle, Gaussian [Frehlich, 1997, Lindelöw-Marsden, 2009]. Here we present the mathematical formulation for a Gaussian pulse. We define  $w_p$  to be the pulse width (standard deviation of the Gaussian pulse), and  $L_p = c\tau$  to be the range gate length of a pulsed lidar, where  $c$  is the speed of light and  $\tau$  is the pulse duration. If we introduce a length scale  $l_p = \sqrt{L_p^2/12 + w_p^2}$

then the same Eq. (3.10) can be used by replacing  $l$  with  $l_p$ , where the  $\Psi(r_1, \beta, \xi)$  function is now given as

$$\Psi(r_1, \beta, \xi) = \frac{3}{2} \Gamma\left(\frac{2}{3}\right) |\cos \xi|^{2/3} \left( {}_1F_1\left(-\frac{1}{3}; \frac{1}{2}; -\frac{r_1^2 \cos^2(\xi + \beta)}{4l_p^2 \cos^2 \xi}\right) - 1 \right), \quad (3.12)$$

where  ${}_1F_1(a; b; x)$  is the Kummer confluent hypergeometric function [Abramowitz and Stegun, 1965]. It is to be noted that using Eqs. (3.10) and (3.12), for some combinations of  $\beta$  and  $r_1/l$  (or  $r_1/l_p$ ),  $\tilde{D}(r_1)$  becomes negative, but Kristensen et al. [2011] provide the range within which Eqs. (3.10) and (3.12) are valid. An advantage of using a pulsed lidar is also that we do not need to apply Taylor's hypothesis in order to compute the separation distance. Thus, instead of using  $r_1$  in Eq. (3.12) we can use the separation distance  $r$  (provided that  $r \ll \mathcal{L}$ ) along the lidar beam, since a pulsed lidar measures at different range gates simultaneously, and hence measure  $\tilde{D}(r)$  along the lidar beam axis [Frehlich, 1997].

### 3.2.2 Six-Beam Scanning

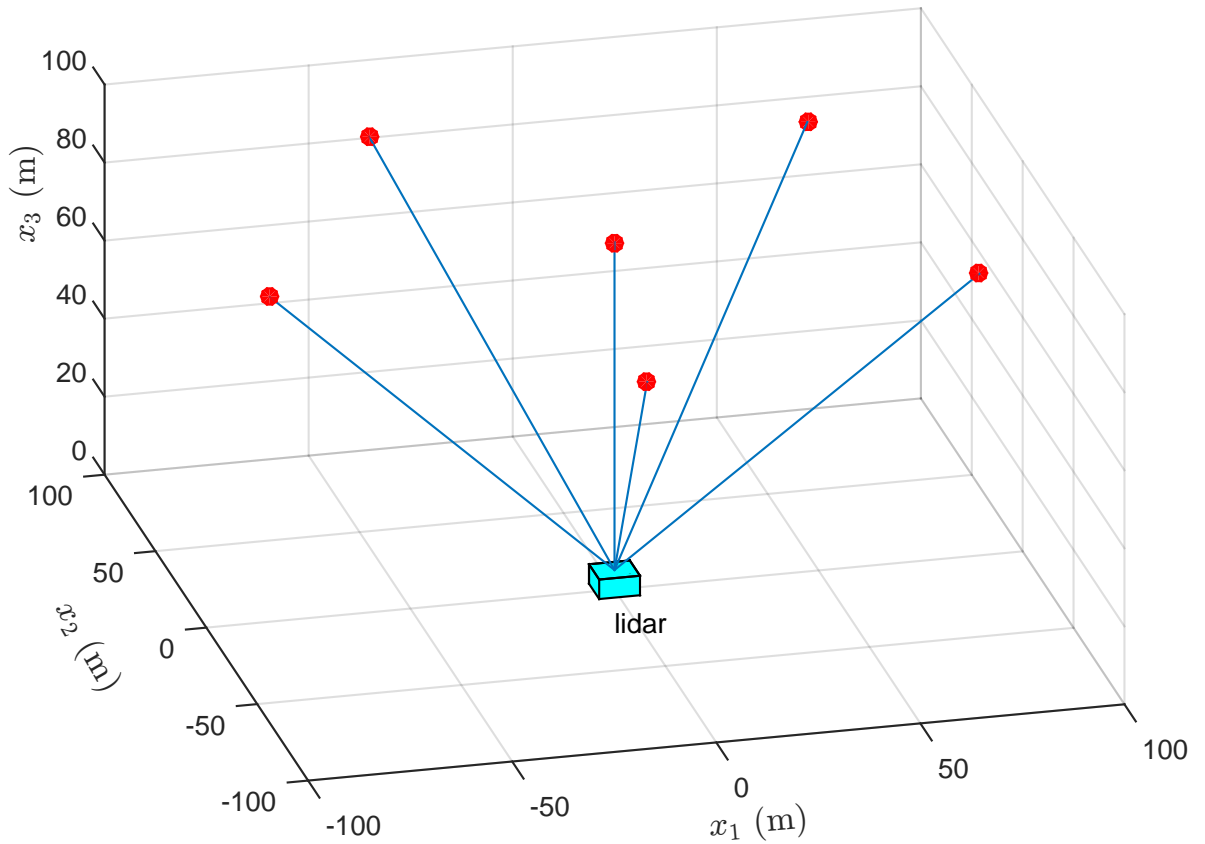


Figure 3.3: Six-Beam Scanning

As seen in section 3.2.1, using only one beam precludes estimation of any turbulence statistics from the lidar data, whereas section 3.1.1 demonstrates that the VAD method of data processing from the scanning lidars result in significant systematic errors. An alternative to the VAD method is the six-beam scanning technique as shown in Fig. 3.3, where two different  $\phi$  are used such that five beams at equally spaced  $\theta$  subtend an angle  $\phi$ , and the

sixth beam is vertical. This is a very recent configuration which has been used to estimate  $\mathbf{R}$  [Sathe et al., 2015].

### 3.2.2.1 Estimating components of $\mathbf{R}$ using six-beam scanning

Instead of using the VAD method to deduce the wind field components for each scan from the lidar data, in this method variances of radial velocities  $\langle v_r'^2 \rangle$  are used. Mathematically it can be represented as,

$$\begin{aligned} \langle v_r'^2 \rangle = & \langle u'^2 \rangle \sin^2 \phi \cos^2 \theta + \langle v'^2 \rangle \sin^2 \phi \sin^2 \theta + \langle w'^2 \rangle \cos^2 \phi \\ & + 2\langle u'v' \rangle \sin^2 \phi \sin \theta \cos \theta + 2\langle u'w' \rangle \sin \phi \cos \phi \cos \theta + 2\langle v'w' \rangle \sin \phi \cos \phi \sin \theta, \end{aligned} \quad (3.13)$$

where  $\langle v_r'^2 \rangle$  is the radial velocity variance. From Eq. (3.13) we can see that for a given  $\theta$  and  $\phi$ , if we have six measurements of  $\langle v_r'^2 \rangle$  then there are six unknowns to be determined, which in a matrix form can be written as,

$$\mathbf{M} \underbrace{\begin{bmatrix} \langle u'^2 \rangle \\ \langle v'^2 \rangle \\ \langle w'^2 \rangle \\ \langle u'v' \rangle \\ \langle u'w' \rangle \\ \langle v'w' \rangle \end{bmatrix}}_{\mathbf{\Sigma}} = \underbrace{\begin{bmatrix} \langle v_{r1}'^2 \rangle \\ \langle v_{r2}'^2 \rangle \\ \langle v_{r3}'^2 \rangle \\ \langle v_{r4}'^2 \rangle \\ \langle v_{r5}'^2 \rangle \\ \langle v_{r6}'^2 \rangle \end{bmatrix}}_{\mathbf{S}}, \quad (3.14)$$

where  $\mathbf{\Sigma}$  is a vector of the components of  $\mathbf{R}$  (because  $\mathbf{R}$  is symmetric, we only need six components),  $\mathbf{M}$  is a  $6 \times 6$  matrix of the coefficients of  $\mathbf{\Sigma}$  that consist of different combinations of  $\theta$  and  $\phi$  (see Eq. 3.13), and  $\mathbf{S}$  is a vector of measurements of  $\langle v_r'^2 \rangle$  at different  $\theta$  and  $\phi$  (where the suffices denote measurements from beam 1 to 6). In principle we can then estimate  $\mathbf{\Sigma}$  using the relation  $\mathbf{\Sigma} = \mathbf{M}^{-1}\mathbf{S}$ , where  $^{-1}$  denotes matrix inverse. It is interesting to know beforehand, whether the measurements from the six beams on only one zenith angle are adequate, i.e. whether we can have six  $\theta$ s and only one  $\phi$ .

From fundamental algebra we understand that Eq. (3.14) will have a finite solution if and only if  $\det \mathbf{M} \neq 0$ , where  $\det$  denotes the determinant of a matrix. In other words  $\mathbf{M}$  should not be a degenerate matrix. From the properties of determinants we know that if any two rows (or columns) of a matrix are identical then its determinant is zero. Also, if the elements of any row (or column) are increased (or decreased) by equal multiples of the corresponding elements of any other row (or column), the value of determinant is unchanged. If we use only one  $\phi$  at different  $\theta$ , and add the first two columns of  $\mathbf{M}$ , we get the first and the third columns of  $\mathbf{M}$  to be multiples of each other, which according to the property of determinants implies  $\det \mathbf{M} = 0$ . Thus  $\mathbf{M}$  becomes degenerate if we use only one  $\phi$ , and thus need  $\langle v_r'^2 \rangle$  measurements from more than one  $\phi$ .

Table 3.1: Optimum six-beam configuration

Beam no.	1	2	3	4	5	6
$\theta(^{\circ})$	0	72	144	216	288	288
$\phi(^{\circ})$	45	45	45	45	45	0

The challenge then is to obtain an optimum combination of  $\theta$  and  $\phi$ . Measured  $\mathbf{S}$  is stochastic, and the random error of  $\mathbf{\Sigma}$  will depend on the particular choice of the  $\theta$ s and  $\phi$ s.



The objective function is chosen such that the sum of the random errors of the components of  $\Sigma$  are minimized and results in the optimum configuration as given in table 3.1.

It is to be noted that using this method, one can only minimize the problem of cross-contamination, but the problem of filtering of small-scale turbulence still remains. Nevertheless it is a much more reliable method than the VAD technique in estimating turbulence statistics [Sathe et al., 2015].

### 3.2.3 Range Height Indicator Scanning

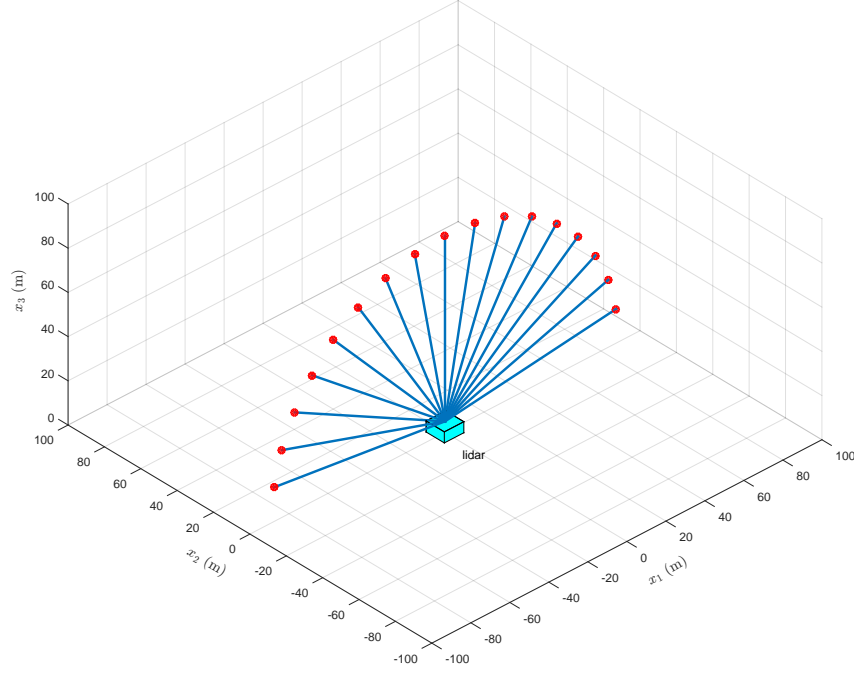


Figure 3.4: Range Height Indicator Scanning

Fig. 3.4 shows the Range Height Indicator (RHI) scanning technique, where the scanning is performed in a vertical ( $x_1 - x_3$ ) plane at different  $\phi$ , but at a constant  $\theta$ . Measurements from different beams at the same height can be used to deduce the wind field components, but it usually requires assumption of horizontal homogeneity over large distances and scanning in at least two vertical planes preferably  $90^\circ$  apart. It is however to be noted that if the wind field components are deduced for each scan and then the statistics are estimated over some averaging interval, it is equivalent to the VAD method of data processing, and the resulting turbulence statistics will be subjected to both, the filtering and the cross-contamination effects as seen for the commercial lidars. Instead using the variances of radial velocities  $\langle v_r'^2 \rangle$  is a much better way of estimating components of  $\mathbf{R}$  [Gal-Chen et al., 1992].

#### 3.2.3.1 Estimating components of $\mathbf{R}$ using the RHI scanning technique

As proposed by Gal-Chen et al. [1992], components of  $\mathbf{R}$  can be estimated from the RHI scanning data if the scanning is performed in the mean wind direction and perpendicular to the mean wind direction. Mathematically it can be represented as,

$$\langle v_r'^2 \rangle = \langle u'^2 \rangle \sin^2 \phi + \langle w'^2 \rangle \cos^2 \phi \pm \langle u'w' \rangle \sin(2\phi), \quad (3.15)$$

for the lidar beam aligned in the mean wind direction. The  $\pm$  sign for  $\langle u'w' \rangle$  indicates whether the wind is blowing away from or towards the lidar beam. Similarly, for the cross-



wind direction we have

$$\langle v_r'^2 \rangle = \langle v'^2 \rangle \sin^2 \phi + \langle w'^2 \rangle \cos^2 \phi \pm \langle v'w' \rangle \sin(2\phi), \quad (3.16)$$

where the  $\pm$  sign indicates positive or negative cross wind beam direction. Equations (3.15) and (3.16) are then solved using the least squares analysis to obtain components of  $\mathbf{R}$  (except  $\langle u'v' \rangle$ ).

It is to be noted that using this method, one can only minimize the problem of cross-contamination, but the problem of filtering of small-scale turbulence still remains. Also it is logistically quite challenging to orient the scanning in the along-wind and cross-wind directions. The lidar would need information of the mean wind direction beforehand in order to orient the scanning pattern in the respective directions.

### 3.2.4 Arc-scanning

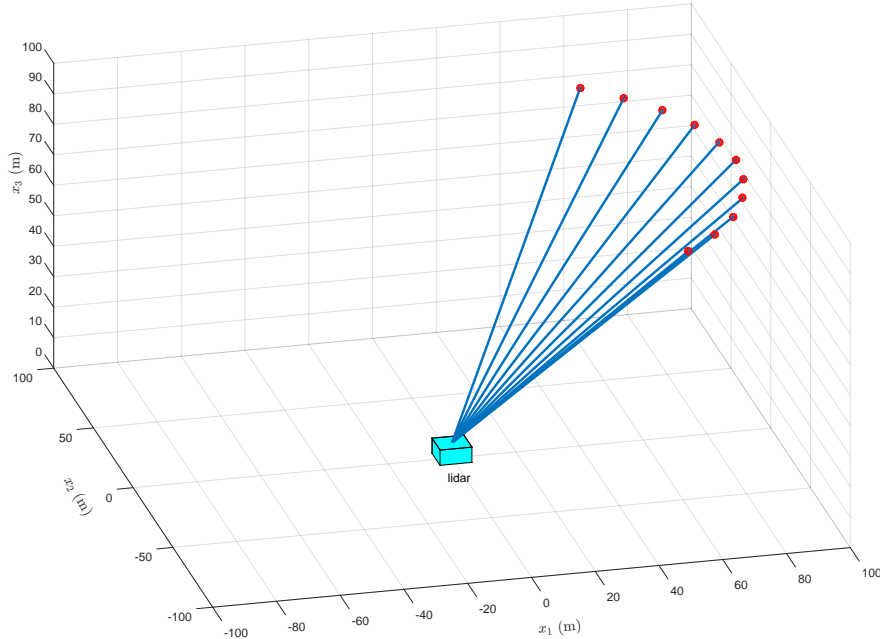


Figure 3.5: Arc Scanning

Fig. 3.5 shows the arc-scanning technique, where similar to the VAD technique of data processing, the wind field components are deduced from the  $v_r$  measurements at different  $\theta$  for a given  $\phi$  (see Eq. 3.1). However the scan is restricted to only an arc, where for example  $\theta$  varies between 0 and 60°, as opposed to 0 and 360° for a full VAD conical scan. One of the obvious advantages of this method is the increase in the sampling frequency, which potentially can capture more (smaller) turbulence scales as compared to the VAD technique. A disadvantage could be increased uncertainty due to random selection of the arc angle within which the measurements are performed.

### 3.2.5 Triple lidar systems - WindScanners

One of the biggest disadvantage of using a single lidar is the necessity of the horizontal homogeneity assumption that almost precludes its use in complex terrains/flows. We are then forced to use a triple lidar system, where the beams cross at a point. Fig. 3.6 shows such

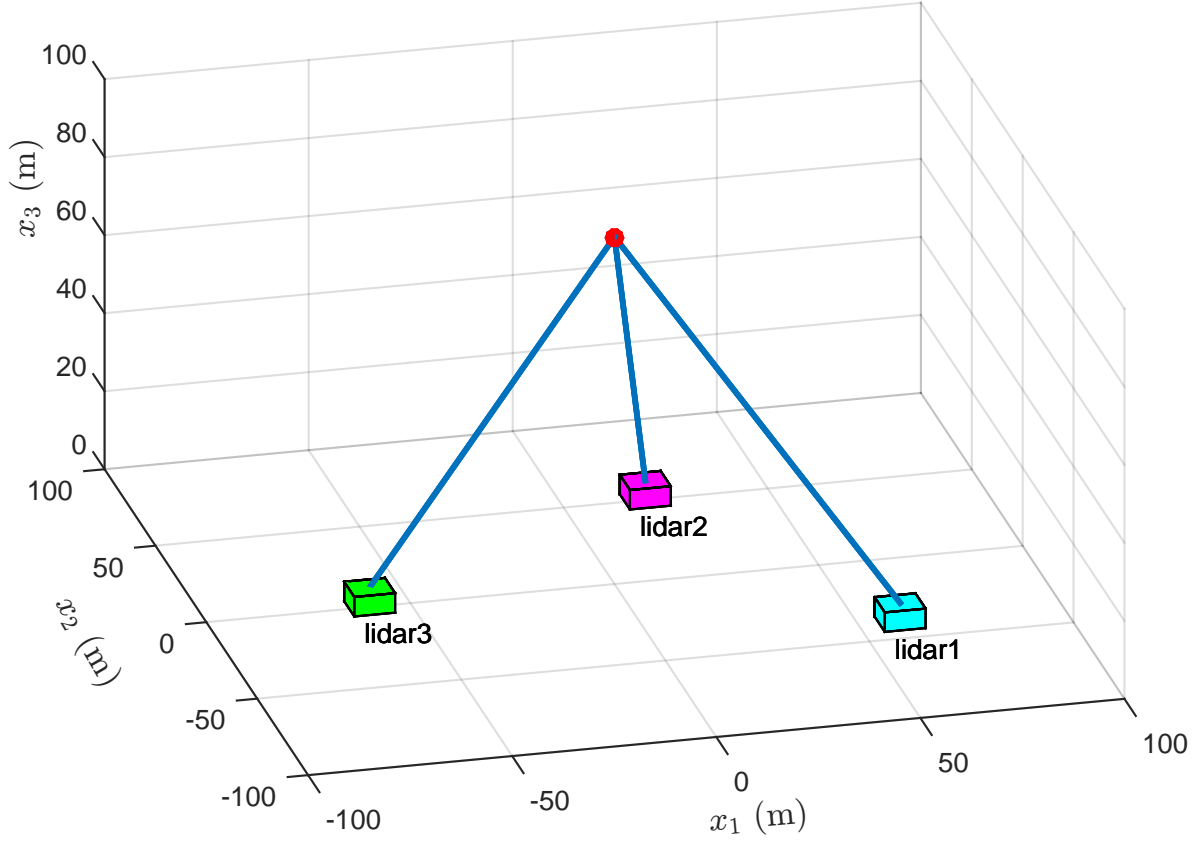


Figure 3.6: Three lidars measuring at one height

a system. The wind field components can be deduced from the  $v_r$  measurements of the three beams. With the ability of lidars to scan in space and meticulous design of synchronizing the beams, measurements from different points in space can be obtained in any kind of terrain/flows.

Deducing the wind field components is then straightforward. If we denote estimates of radial velocities from the three beams as a vector  $\mathbf{v}_r = (v_{r1}, v_{r2}, v_{r3})$ , and  $\mathbf{N}$  as a  $3 \times 3$  matrix of trigonometric coefficients (consisting of  $\phi$  and  $\theta$ ) of the wind field components then from Eq. (3.1)  $\mathbf{v}$  can be written as,

$$\mathbf{v} = \mathbf{N}^{-1} \mathbf{v}_r \quad (3.17)$$

### 3.2.6 Dual lidar systems

Fig. 3.7 shows the dual lidar system, where only two lidars are used to deduce two wind field components. Usually it is assumed that the vertical component ( $w$ ) is very small compared to the horizontal components ( $u$  and  $v$ ), but it requires very small elevation angles  $\alpha$  (and consequently large scanning distances) as can be seen in Fig. 3.7 [Newsom et al., 2015]. Such a system can be quite useful in scanning an offshore wind field, where the three lidar systems may not be practical to use. Also an additional savings in the costs can be achieved by getting rid of one lidar.

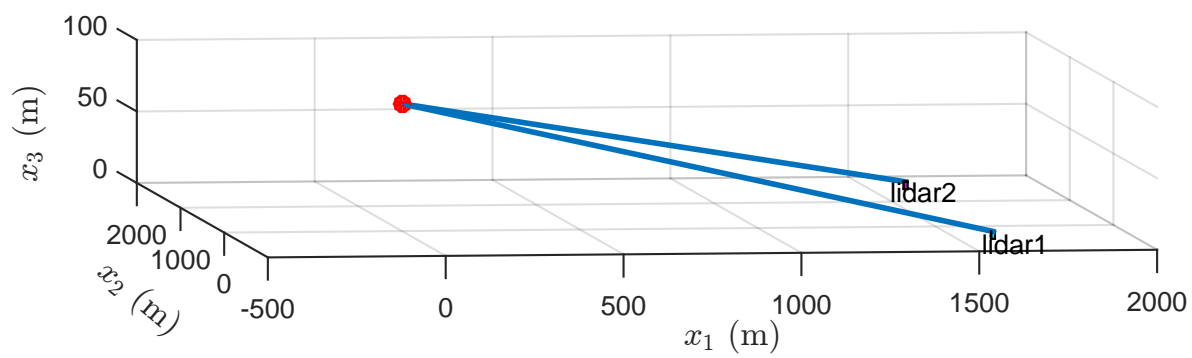


Figure 3.7: Two lidars measuring at one height

## Chapter 4

# State-of-the-art

Estimation of the turbulence statistics using lidars has been a topic of research since the 1970s, but there has been a significant growth in lidar turbulence studies since the mid 1990s. Several measurement configurations as described in chapter 3 have been used in the past studies. [Sathe and Mann \[2013\]](#) summarize these studies, where mathematical formulations of the deduced turbulence statistic is also presented. In this chapter we only classify the turbulence studies based on the estimated turbulence statistic defined in chapter 1. A reader who is interested in mathematical formulations is referred to section 4.1 of [Sathe and Mann \[2013\]](#).

Until the mid- and late 1990s, the focus was more on developing new data-processing methods to extract turbulence information. New algorithms for efficiently processing the raw lidar data are still being developed, as seen in the recent work by [Mann et al. \[2010\]](#). Nevertheless, many studies have benefited from the continuous developments in the past, where simulation studies and measurement campaigns have been carried out. Because lidar is not yet an established technology to measure atmospheric turbulence, it is important to compare lidar measurements with a reference instrument, as emphasized in the review article by [Wilczak et al. \[1996\]](#). In their review, lidar technology was termed to be a “young adult” in comparison to sodars and radars. With the recent spurt in the measurement campaigns using lidars, we think that it has grown beyond its status of “young adult”.

Table 4.1 groups the studies that have focused on estimation of turbulence quantities using either simulation or lidar measurements. For each turbulence quantity, the total number of studies is also given. It is evident that significant effort has been focused on estimation of  $\varepsilon$ , followed by  $R_{ij}$ ,  $\ell_{ij}$ , outer length scale of turbulence  $\mathcal{L}$ , radial velocity variance  $\langle v_r'^2 \rangle$ , filtered radial velocity structure function  $\tilde{D}(r)$  for a separation distance  $r$ , filtered radial velocity spectrum  $\tilde{F}(k_1)$ , and  $F_{ij}(k_1)$ .

Table 4.1: Grouping of the past studies according to the estimated turbulence quantity using a lidar.

No.	Quantities Estimated	List of references	Total
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1	Turbulent kinetic energy dissipation rate, $\varepsilon$	Banakh and Smalikho [1997a,b], Banakh and Werner [2005], Banakh et al. [1995b, 1996, 1997, 1999, 2010], Chan [2011], Collier et al. [2005], Davies et al. [2004, 2005], Davis et al. [2008], Dors et al. [2011], Drobinski et al. [2000], Frehlich [1997], Frehlich and Cornman [2002], Frehlich and Kelley [2008], Frehlich et al. [1994, 1998, 2006, 2008], Gal-Chen et al. [1992], Kristensen et al. [2011, 2012], Kunkel et al. [1980], Lothon et al. [2009], O'Connor et al. [2010], Smalikho et al. [2005]	29
2	Components of the auto-covariance matrix, $R_{ij}$	Banta et al. [2006], Cohn et al. [1998], Collier et al. [2005], Davies et al. [2003, 2005], Davis et al. [2008], Drobinski et al. [2004], Eberhard et al. [1989], Frehlich et al. [1998], Gal-Chen et al. [1992], Kunkel et al. [1980], Lang and McKeogh [2011], Mann et al. [2010], Pichugina et al. [2008], Sathe et al. [2011b], Tucker et al. [2009], Wagner et al. [2009]	17
3	Integral turbulent length scale $\ell_{ij}$ , outer scale of turbulence $\mathcal{L}$	Banakh and Werner [2005], Banakh et al. [1999], Cohn et al. [1998], Collier et al. [2005], Davies et al. [2004, 2005], Drobinski et al. [2000], Frehlich [1997], Frehlich and Cornman [2002], Frehlich and Kelley [2008], Frehlich et al. [1998, 2006, 2008], Lothon et al. [2006, 2009], Smalikho et al. [2005]	16
4	Radial velocity variance, $\langle v_r'^2 \rangle$	Banakh and Werner [2005], Branlard et al. [2013], Davies et al. [2004], Drobinski et al. [2000], Eberhard et al. [1989], Frehlich [1997], Frehlich and Kelley [2008], Frehlich et al. [1998, 2006, 2008], Gal-Chen et al. [1992], Mayor et al. [1997]	12
5	Filtered radial velocity spectrum, $\tilde{F}(k_1)$	Angelou et al. [2012], Banakh et al. [1997, 1999], Davies et al. [2004], Dors et al. [2011], Drobinski et al. [1998, 2000], Frehlich et al. [1998], Kristensen et al. [2011], Mann et al. [2009], Mayor et al. [1997], Sjöholm et al. [2009]	12
6	Filtered radial velocity structure function, $\tilde{D}(r)$ (or $\tilde{D}(r_1)$ )	Banakh and Smalikho [1997b], Banakh et al. [1999, 2010], Chan [2011], Davies et al. [2004], Frehlich [1997], Frehlich and Cornman [2002], Frehlich et al. [1994, 1998, 2008], Kristensen et al. [2011, 2012]	12
7	One-dimensional spectrum of the components of the wind field, $F_{ij}(k_1)$	Canadillas et al. [2010], Davies et al. [2005], Drobinski et al. [2004], Hardesty et al. [1982], Lawrence et al. [1972], Lothon et al. [2009], O'Connor et al. [2010], Sathe and Mann [2012]	8
8	Third order moments $\langle w'^3 \rangle$	Cohn et al. [1998], Gal-Chen et al. [1992], Lenschow et al. [2000]	3

9	Kinematic heat flux, $\langle w'\theta' \rangle$	<a href="#">Davis et al. [2008]</a> , <a href="#">Gal-Chen et al. [1992]</a>	2
10	Coherence of the components of the wind field, $\text{coh}_{ij}(k_1)$	<a href="#">Kristensen et al. [2010]</a> , <a href="#">Lothon et al. [2006]</a>	2

#### 4.1 $\varepsilon$ , $\tilde{F}(k_1)$ , $\tilde{D}(r)$

The greatest advantage of estimation of  $\varepsilon$  is that we can exploit the universal behavior of isotropy in the inertial subrange, either in the Fourier domain (using velocity spectrum) or the temporal domain (using structure function) [[Pope, 2000](#)]. Thus, estimation of  $\varepsilon$  involves estimation of either  $\tilde{F}(k_1)$  or  $\tilde{D}(r)$  in the inertial subrange from the lidar beam that is oriented in any direction. The associated challenges are then threefold: proving the existence of the inertial subrange, identifying the inertial subrange from the lidar data, and averaging inside the probe volume. From [Pope \[2000\]](#), we understand that in order to have a well-defined inertial subrange, we need large Reynolds number flows. Fortunately, atmospheric flows are usually characterized by large Reynolds numbers [[Wyngaard, 2010](#)], especially during convective daytime conditions. Stable atmospheric conditions that normally occur during the late night and early morning, can however present challenges since they are associated with low Reynolds number turbulence [[Wyngaard, 2010](#)]. We can then assume that inertial subrange is well defined for most of the day, except during late night and early morning conditions.

The challenge associated with identifying the inertial subrange from lidar measurements is mainly due to the probe length of a lidar. In principle, we need only one measurement – of either  $\tilde{F}(k_1)$  or  $\tilde{D}(r)$  – in the inertial subrange. However, in order to avoid statistical uncertainty, it is recommended that one take multiple measurements, and fit a model to these measurements. From [Mann et al. \[2009\]](#), [Sjöholm et al. \[2009\]](#) and [Sathe \[2012\]](#), it is clear that due to the probe length of a lidar, most of the turbulence scales in the inertial range are filtered out. Modeling the lidar filter function then becomes inevitable, which has fortunately been carried out by [Smalikho \[1995\]](#), [Banakh et al. \[1996\]](#), [Frehlich \[1997\]](#), [Smalikho et al. \[2005\]](#), [Sjöholm et al. \[2009\]](#) and [Mann et al. \[2009\]](#). In [Sjöholm et al. \[2009\]](#) and [Mann et al. \[2009\]](#), the goal was only to compare the lidar volume-averaged measurements of the radial velocity spectrum with reference point measurements; an estimation of  $\varepsilon$  was not carried out. These studies could be extended further to estimate  $\varepsilon$  by using the isotropic or anisotropic form of spectral tensor with a given energy spectrum. Apart from the filtering effect, we also need to identify the cut-off low wavenumber range when using  $\tilde{F}(k_1)$ , and the maximum separation distance when using  $\tilde{D}(r)$  in order to identify the inertial subrange.

In summary, there are four ways of estimating  $\varepsilon$ : width of the Doppler spectra ([Smalikho, 1995](#); [Banakh et al., 1995a, 2010](#); [Smalikho et al., 2005](#)), radial velocity spectrum [[Banakh and Smalikho, 1997a](#), [Banakh et al., 1995b, 1997](#), [Collier et al., 2005](#), [Davies et al., 2005](#), [Davis et al., 2008](#), [Dors et al., 2011](#), [Drobinski et al., 2000](#), [Gal-Chen et al., 1992](#), [Kristensen et al., 2011](#), [Lothon et al., 2009](#), [O'Connor et al., 2010](#)], line-of-sight radial velocity structure function [[Banakh and Smalikho, 1997a,b](#), [Banakh and Werner, 2005](#), [Banakh et al., 1999](#), [Davies et al., 2004](#), [Frehlich, 1997](#), [Frehlich and Cornman, 2002](#), [Frehlich et al., 1994, 1998](#), [Smalikho et al., 2005](#)], and radial velocity azimuthal structure function [[Banakh and Smalikho, 1997a](#), [Banakh et al., 1996, 1999](#), [Chan, 2011](#), [Frehlich and Kelley, 2008](#), [Frehlich et al., 2006, 2008](#), [Kristensen et al., 2012](#)]. Very few studies have exploited the Doppler spectral width to estimate  $\varepsilon$ ; the reasons could be that for a C-W lidar the applicability of the Doppler spectral width is limited to  $l \ll \mathcal{L}$ , and for a pulsed lidar it is quite complicated to process the data

[Smalikho et al., 2005]. Nevertheless, as shown by Banakh et al. [2010], for a pulsed lidar it could be advantageous to use the Doppler spectral width approach, since the random errors in  $\varepsilon$  can be reduced at higher turbulence levels than they can in the structure function approach, or equally, using the radial velocity spectrum approach.

## 4.2 $\langle v_r'^2 \rangle$ , $\ell_{ij}$ , $\mathcal{L}$

Apart from  $\varepsilon$ , another important parameter that characterizes turbulence is the length scale. The two most commonly used definitions of the length scale are  $\ell_{ij}$  and  $\mathcal{L}$ , which have physically different interpretations.  $\mathcal{L}$  (also called the outer length scale of turbulence) is the length scale corresponding to the maximum spectral energy, whereas  $\ell_{ij}$  can be interpreted as the length scale up to which turbulence is correlated. The two scale lengths can, however, be shown to be related to each other, as was done by Frehlich and Cornman [2002], Smalikho et al. [2005] and Lothon et al. [2006]. Thus,  $\ell_{ij}$  can be estimated from its relationship with  $\mathcal{L}$  [Collier et al., 2005, Davies et al., 2004, Frehlich and Cornman, 2002, Frehlich et al., 2006, Lothon et al., 2006, 2009, Smalikho et al., 2005], or by using the definition given in Eq. (1.4) [Cohn et al., 1998]. Practically,  $\ell_{ij}$  is estimated from the values of the autocorrelation function at the first zero crossing, but Davies et al. [2005] estimated the same using some properties of the autocorrelation function.  $\mathcal{L}$  can be estimated using the structure function approach [Frehlich, 1997, Frehlich and Kelley, 2008, Frehlich et al., 1998, 2008]. Drobinski et al. [2000] followed a slightly different approach, in which the radial velocity spectrum is split into two regions; one is the energy-containing range, and the other contains the inertial subrange up to the dissipation range. Measurements of the radial velocity spectrum can thus be fitted to this model and  $\mathcal{L}$ ,  $\varepsilon$  estimated simultaneously. Interestingly, Banakh et al. [1999] and Banakh and Werner [2005] also use the term outer length scale for  $\ell_{ij}$ , but we believe that it is important to distinguish between the two length scales.

Fewer studies have been carried out to estimate  $\langle v_r'^2 \rangle$  than to estimate  $\varepsilon$  (see Table 4.1). This is perhaps because information of all turbulence scales is required to estimate  $\langle v_r'^2 \rangle$ , and a universal isotropic relation does not suffice. Although Eberhard et al. [1989] and Gal-Chen et al. [1992] have estimated  $\langle v_r'^2 \rangle$  from lidar measurements, no consideration to probe volume averaging was given, and thus any other turbulence statistic derived using these measurements would not contain information on small-scale turbulence. All subsequent studies have followed the pioneering work of Frehlich [1997], in which information about small-scale turbulence was recovered by modeling the filter function. The main contributions of the Frehlich [1997] method are first, that it presents a technique to derive expressions of the radial velocity structure function (or, equivalently, the radial velocity spectrum) for a lidar pulse with any given shape; second, it presents a turbulence model, with which we can estimate  $\langle v_r'^2 \rangle$  and  $\ell_{ij}$ . One can thus use a non-Gaussian shape for the pulse and derive a different functional form of the spatial filter [Davies et al., 2004], or use a different turbulence model, e.g. von Kármán [1948] isotropic spectral tensor model [Frehlich and Cornman, 2002], or a more realistic anisotropic Mann [1994] spectral tensor instead of the empirical Kaimal et al. [1972] models [Frehlich, 1997, Frehlich et al., 1998]. Using  $\hat{D}(r)$  to estimate  $\langle v_r'^2 \rangle$  from a pulsed lidar has the limitation of coarse vertical resolution. An azimuthal structure function approach can then be used to improve the vertical resolution [Banakh et al., 1996, Frehlich and Kelley, 2008, Frehlich et al., 2006, Kristensen et al., 2012]. Without using any turbulence model, Mann et al. [2010] suggested a technique (only for C-W lidars) to estimate  $\langle v_r'^2 \rangle$  using the mean Doppler spectrum. The validity of this technique is successfully demonstrated in Branlard et al. [2013].



### 4.3 $R_{ij}$ , $F_{ij}(k_1)$

$R_{ij}$  is one of the most important turbulence statistics used in the wind energy industry, due to the use of  $\langle u'^2 \rangle$  in the definition of turbulence intensity (see Eq. 1.3). Unfortunately, it is also one of the most challenging statistics to obtain from the lidar data, partly due to challenges in data processing, and partly due to economic reasons. If economics is not a major constraint, then three lidars with beams intersecting at one point will provide spatially filtered turbulence statistics [Mann et al., 2009]. With two lidars, we are restricted to estimating the turbulence statistics of only two components, i.e., horizontal and vertical [Collier et al., 2005, Davies et al., 2005].

Normally, the economics of a project are important and we are then restricted to using only one lidar. In this case, a lidar beam can be oriented in the direction of the turbulence statistic that we are interested in estimating. For example, if we are interested in estimating  $\langle u'^2 \rangle$ , then ideally the lidar beam should be pointed horizontally in the mean wind direction at the height of interest, and for the period within which  $\langle u'^2 \rangle$  is obtained [Lawrence et al., 1972]. For a ground-based lidar system this would be impossible since the beam would only measure wind that is very close to the ground. Alternatively, we could point the lidar beam at a very small elevation angle and assume that the contributions from the vertical velocity are negligible [Banta et al., 2006, Collier et al., 2005, Drobinski et al., 2004, Pichugina et al., 2008]. An open question then is, how small the elevation should be so that the vertical velocity contributions can be neglected? Drobinski et al. [2004], Banta et al. [2006], and Pichugina et al. [2008] neglected the vertical velocity contributions up to an elevation angle of  $20^\circ$ , but provided no justification for the assumption of negligible vertical velocity contributions. This method also requires that the horizontal homogeneity assumption is valid over a larger area, particularly if we are interested in measuring turbulence statistics at greater heights and/or several heights. Measurements of  $\langle w'^2 \rangle$  can be relatively easier to take, since we only need to point the beam in the vertical direction [Cohn et al., 1998, Tucker et al., 2009]. In principle, following Frehlich [1997] and Banakh and Smalikho [1997b] approach, we can then obtain unfiltered  $\langle w'^2 \rangle$  from  $\langle v_r'^2 \rangle$ .

$R_{ij}$  can also be obtained using scanning lidar data, either using RHI scanning [Davies et al., 2003, Davis et al., 2008, Gal-Chen et al., 1992] or VAD scanning [Eberhard et al., 1989, Mann et al., 2010]. If, say for a VAD scanning, we use high-frequency  $v_r$  measurements, deduce the  $u$ ,  $v$ , and  $w$  components at every measurement time step, and obtain, say,  $\langle u'^2 \rangle$  or  $F_{11}(k_1)$ , then apart from the probe volume averaging effect, large systematic errors will also be introduced in the measurement of  $\langle u'^2 \rangle$  due to the contamination by the diagonal and cross components of  $\mathbf{R}$  [Sathe and Mann, 2012, Sathe et al., 2011b]. In such cases, one should be very careful in using the  $R_{ij}$  measurements obtained from a scanning lidar, since removing only the probe volume filtering effect [Wagner et al., 2009] without giving consideration to cross-contamination, or neglecting the effects of systematic errors completely [Lang and McKeogh, 2011] will provide erroneous values. Using  $\langle v_r'^2 \rangle$  instead of high frequency  $v_r$  measurements to obtain  $R_{ij}$  is then essential in order to avoid contamination by the components of  $\mathbf{R}$  [Eberhard et al., 1989, Gal-Chen et al., 1992, Mann et al., 2010, Sathe, 2012]. The cross-contamination effect is minimized using the six-beam method, but compensating for the spatial averaging effects for pulsed lidars still remains a challenge. Experimental evidence suggests that the six-beam method partly overcomes the problem of significant probe volume averaging that is otherwise observed by the VAD method [Sathe et al., 2015]. The unfiltered  $\langle v_r'^2 \rangle$  can be obtained using methods suggested by Frehlich [1997] and Mann et al. [2010], and hence unfiltered  $R_{ij}$  will also be obtained.

Estimating  $F_{ij}(k_1)$  from lidar data is even more challenging than estimating  $R_{ij}$ , since we need high frequency measurements of  $v_r$ . For a scanning lidar, combining high frequency



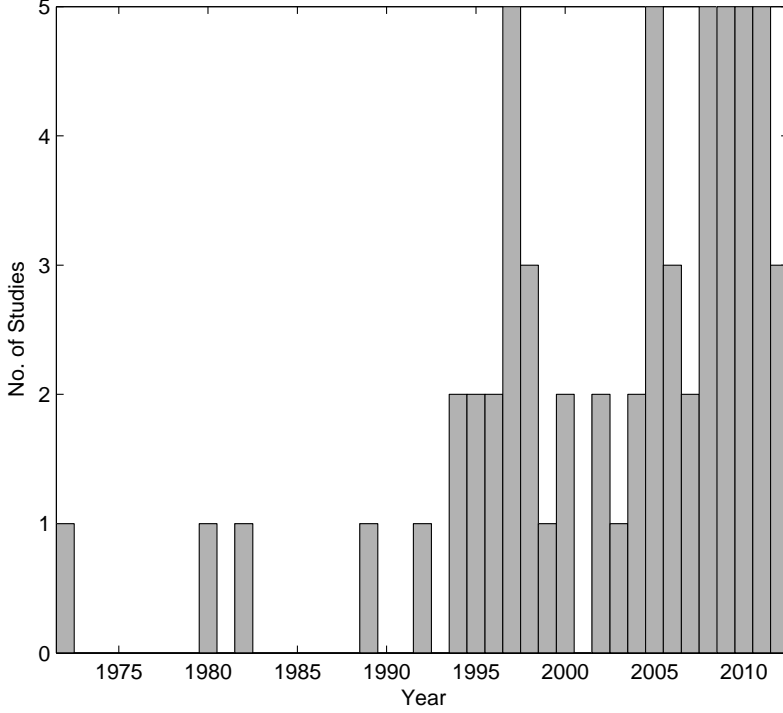


Figure 4.1: Number of studies per year on lidar turbulence measurements.

measurements from the lidar beams oriented in different directions (VAD method) results in erroneous measurements of  $F_{ij}(k_1)$  [Canadillas et al., 2010, Sathe and Mann, 2012]. Most studies in the past have thus used either a staring lidar configuration [Davies et al., 2005, Lawrence et al., 1972, Lothon et al., 2009, O'Connor et al., 2010], or neglected contributions from the  $w$  component at small elevation angles [Drobinski et al., 2004, Hardesty et al., 1982].

#### 4.4 $\langle w'^3 \rangle$ , $\langle w'\theta' \rangle$ , $\text{coh}_{ij}(k_1)$

Very little effort has been focused on the estimation of the third order moment  $\langle w'^3 \rangle$ ,  $\langle w'\theta' \rangle$  and  $\text{coh}_{ij}(k_1)$ . One of the reasons could be the complexity of data processing and the associated errors that present great challenges in their estimations. Particularly, an estimation of  $\langle w'\theta' \rangle$ , requires not only an estimation of  $\varepsilon$ , but also requires estimation of either  $\langle w'^3 \rangle$  [Gal-Chen et al., 1992], or  $\langle w'^2 \rangle$  [Davis et al., 2008]. Estimating higher order moments, particularly third and fourth order, introduce large errors in the measurements [Lenschow et al., 1994, 2000]. Fortunately, we can reduce the errors in higher moments using the autocorrelation technique [Lenschow et al., 2000] or the spectral technique [Frehlich et al., 1998], which increase the potential of estimating the heat flux using the [Gal-Chen et al., 1992] method.

#### 4.5 Summary

Figure 4.1 summarizes the number of studies that have significantly contributed to the research on turbulence measurements using wind lidars from 1972–2012. Research on lidar turbulence measurements dates back to 1972, but it was not until 1997 that the publication rate picked up pace. If we consider that the lidar turbulence measurement research encom-

passes the period 1972–2012, then more than 80 % of the research was carried out in the latter half of the 40 year period, i.e., from 1997–2012. In the first 25 years of development, barring the works of [Smalikho \[1995\]](#) and [Banakh et al. \[1996\]](#), focus was more on extracting turbulence information without taking into account probe volume averaging. Since then substantial effort has been put into modeling the averaging effect inside the lidar probe volume, mainly by Professor V. A. Banakh and Dr. I. N. Smalikho from the V. E. Zuev Institute of Atmospheric Optics of Russian Academy of Sciences, Siberian Branch, Russia, and the late Dr. R. Frehlich from the University of Colorado, USA. Interestingly, these scientists pioneered new processing algorithms independently of each other during roughly the same period, i.e. from the mid 1990s until the mid 2000s, wherein they demonstrated how to extract unfiltered turbulence parameters [[Banakh and Smalikho, 1997b](#), [Banakh et al., 1996](#), [Frehlich, 1997](#), [Frehlich et al., 2006](#), [Smalikho et al., 2005](#), [Smalikho, 1995](#)]. We believe that this development has significantly contributed to the number of research studies carried out in the last 15 years. Further development in processing algorithms will also greatly benefit from their works. We expect that the number of such studies will continue to increase due to increase in wind-energy development all over the world.

## Chapter 5

# Experimental Evidence of the Estimated Turbulence Statistics from Lidar Measurements

In this chapter we provide details of some of the studies and experimental campaigns using lidars that have been carried out at a test center in Denmark with a focus on estimated turbulence statistics. Different methods of post processing the lidar data have been applied, and a variety of turbulence statistics estimated. Amongst others the commonly estimated turbulence statistics are the components of  $\mathbf{R}$  and  $F_{ij}(k_1)$ . It is usually not straightforward to report different studies in a consistent manner. Therefore the reporting is structured such that for each study, at first the objective is stated clearly followed by the site and instrument details. Subsequently the illustrations of the mean and turbulence statistics are provided. In many studies the estimated turbulence statistics are compared with those estimated by some reference instrument such as a cup or a sonic anemometer. It is therefore important to know beforehand the measurement uncertainty of the reference instrument itself. According to JCG [2008], measurement uncertainty can be quantified as the dispersion of the measurand. Owing to a lack of a standard procedure, it is not straightforward to estimate the measurement uncertainty of the sonics, but is usually available for cup anemometers. Therefore where available they are stated along with the measurement uncertainty of the lidars used. It is to be noted that although the studies are numbered sequentially they are not sorted according to the dates of the experimental campaigns.

Because modern day lidars are known to measure the mean wind speeds as accurately and precisely as the reference cup/sonic anemometers, the illustrations of the mean wind speed comparisons provide an initial check that further provide reasonable confidence in the estimated turbulence statistics. In other words if the comparisons between the mean wind speeds estimated using a lidar and those estimated using a reference instrument is quite poor then there is very little reason to trust the estimation of the turbulence statistics for that experimental campaign.

### 5.1 Study 1

#### 5.1.1 Introduction

The objective of this study was to understand the estimation of the components of  $\mathbf{R}$  (see Eq. 1.2), where the VAD method (see section 3.1.1) was used in post processing the lidar data. Modelling of the estimated  $\mathbf{R}$  using the VAD method was carried out, and measurements

from two (a CW and a pulsed) commercial lidars were used to verify the model. Comparisons of the estimated  $R_{ij}$  from the model and the data were also carried out with those estimated by reference sonic anemometers at different heights. For details on the model the reader is referred to [Sathe et al., 2011b].

### 5.1.2 Measurement details

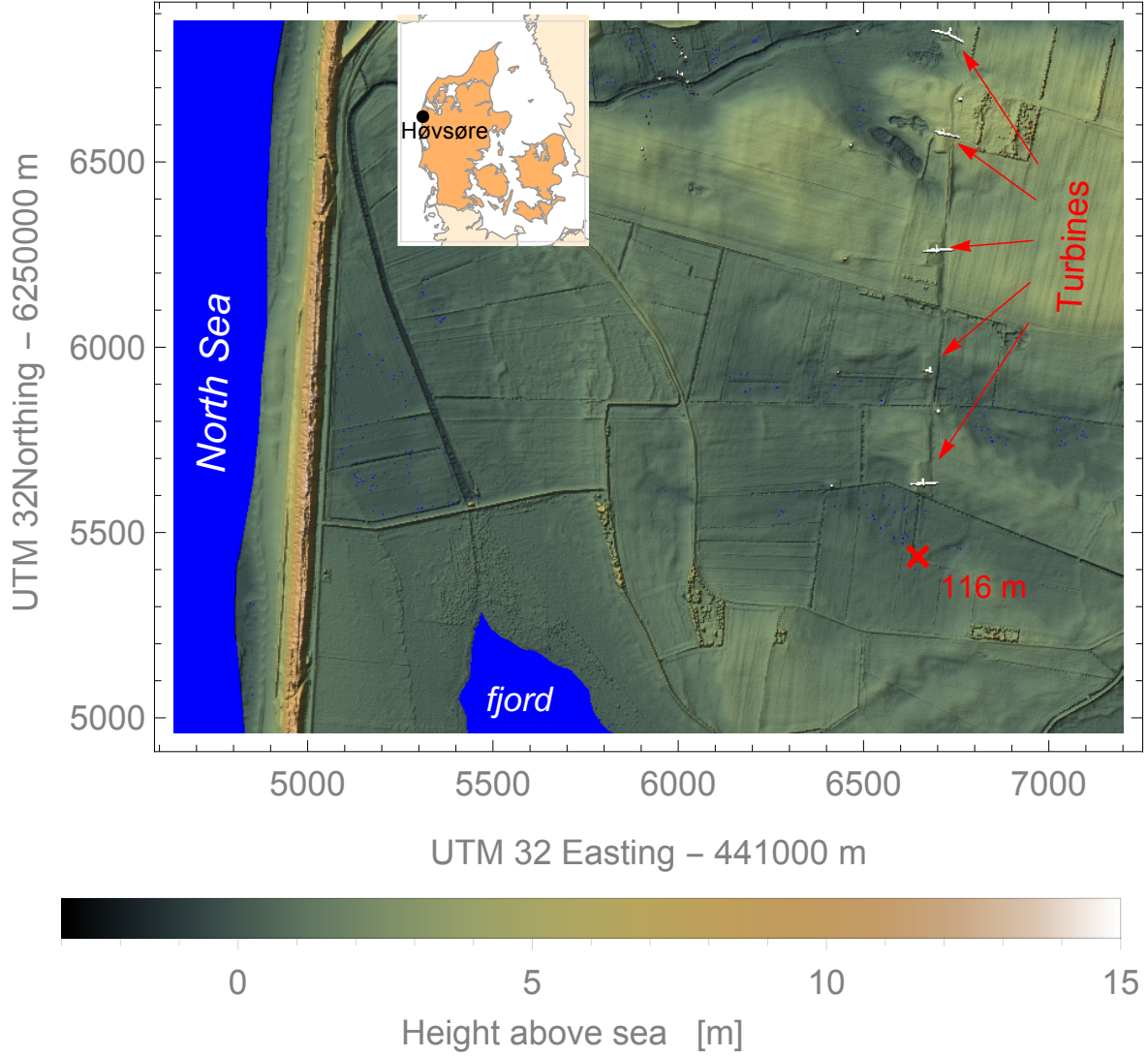


Figure 5.1: Location of the test site at Høvsøre, Denmark

The measurements were performed at the Danish National Test Center for Large Wind Turbines at Høvsøre, Denmark. Figure 5.1 shows the location of the test center in Denmark (see inset in Fig. 5.1). The site is about 1.7 km east from the North Sea at a mean height of 2 m above mean sea level. The Nissum Fjord lies  $\approx 800$  m south of the station's 116.5 m tall met mast. The site also comprises five turbine stands, which turbine manufacturers rent for the machine's testing and research. Besides this, (not shown in Fig. 5.1) there are five power curve masts, two lighting towers and a central service building. The eastern sector is characterized by a relatively flat homogeneous terrain.

The lidar measurements were used from two different types of commercial lidars, a CW ZephIR lidar and a pulsed WindCube lidar. The reference measurements were used from

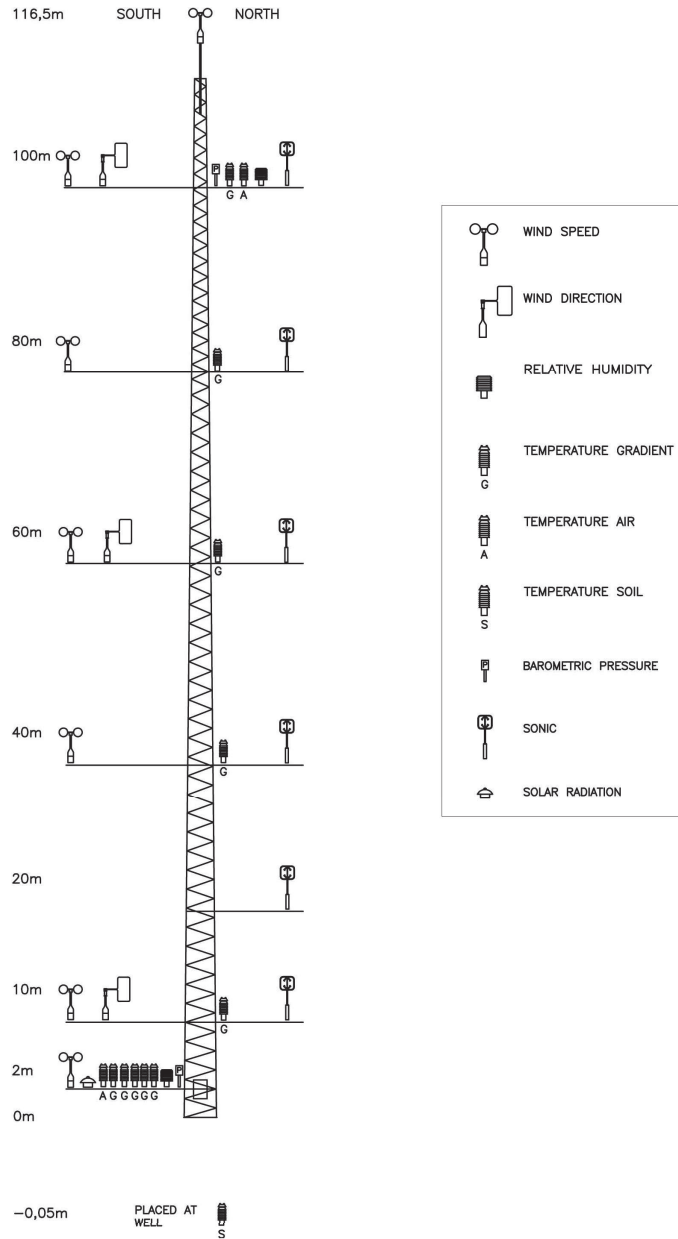


Figure 5.2: Sketch of the instruments on the 116.5 m met mast at Høvsøre

the sonic anemometers installed at different heights on a 116.5 m mast, also known as the Høvsøre met mast (see Fig. 5.2). The mast is an equilateral triangular lattice structure, where one of the vertices of the triangle points east and one of the bases of the triangle points in the North-South direction. The width of the mast decreases from 7.15 m at the ground to 1.10 m at 100.5 m. The sonic anemometers are installed on slender booms pointing North, whereas the cup anemometers and wind vanes are installed on the booms pointing South. In order to further avoid the influence of the wakes from the wind turbines and the met mast on lidar measurements, and inhomogeneities due to the sudden change of roughness (sea-land transition, see Fig. 5.1), only data periods with easterly winds ( $50^{\circ}$ – $150^{\circ}$ ) are analyzed. Table 5.1 provides some details of the experiment.

Table 5.1: Instrument and Measurement Details of Study 1

	Reference Met Mast (Sonics)	CW Lidar, ZephIR	Pulsed lidar, WindCube
Location, UTM zone 32V WGS84 datum	447647 m, E and 6255435 m, N	447635 m, E and 6255467 m, N	447648 m, E and 6255439 m, N
Model/Version	Metek USA1 F2901A	v1	v1
Period of Measurement	Corresponding to the respective li- dar	April–November 2009	January–April 2009
Sampling rate (Hz)	20	$\approx 1$	$\approx 0.667$
Averaging Period (min)	10		
Measurement Heights (m)	40, 60, 80 and 100		

### 5.1.3 Mean wind speed comparisons

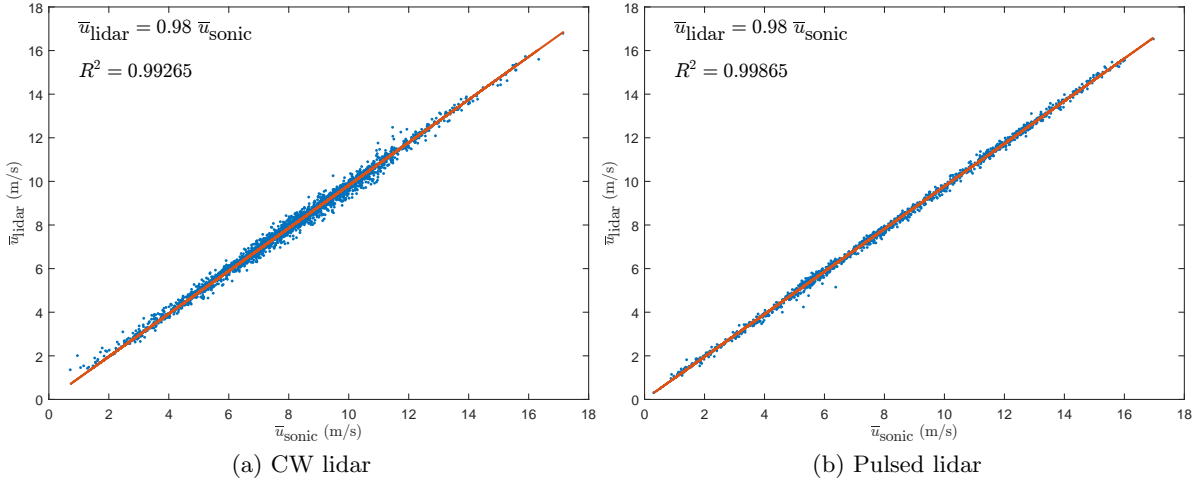


Figure 5.3: Comparing the mean wind speeds with a reference sonic anemometer at 100 m

Fig. 5.3 shows the comparison of the mean wind speeds estimated from lidar measurements and those estimated from a reference sonic at 100 m. For both types of lidars (CW and pulsed) the systematic error (characterized by the slope of the linear curve fit), and the uncertainty (characterized by the coefficient of determination  $R^2$ ) are negligible. The closer the value of the slope to one, the smaller the systematic error, whereas the closer the value of  $R^2$  to one, the smaller the uncertainty. Since for both types of lidars the systematic errors and uncertainties with respect to a reference sonic are  $\approx 2\%$  and  $\lesssim 1\%$  only, it gives enough confidence to proceed with the turbulence analysis.

### 5.1.4 Turbulence measurements

Owing to the availability of the high frequency (time series) data of  $v_r$  and  $\mathbf{v}$  for both types of lidars and sonics respectively at different heights, variances of the  $u$ ,  $v$  and  $w$  components were estimated for several 10-min periods. The instrument and measurement details are given in table 5.1. The choice of the 10-min averaging period is driven by the common practice in the wind energy industry to use 10-min first- and second-order statistics of the wind field

components [IEC, 2005]. Ideally the choice of an averaging period should be governed by the turbulent structure, i.e. the integral length/time scale of a turbulent time series [Lenschow et al., 1994] (see section 1 and Eq. 1.4 for the definition of integral scales). However, in practice it is very difficult to implement such a technique. Therefore a compromise is usually made with regards to the choice of the averaging period, which is taken as 10-min in this experimental campaign.

From the lidar time series, the VAD method of post processing the data is implemented and the variances of the  $u$ ,  $v$  and  $w$  components are estimated for every 10-min period. Simultaneously the corresponding statistics are also estimated using the sonics, which are used as reference measurements. For the respective measurement periods (depending on the type of lidar), several 10-min statistics were obtained representing different ensembles. In order to incorporate variation of the turbulent structure under different atmospheric stabilities, Obukhov length  $L$  (see Eq. 1.11) was estimated using the sonic measurements at 20 m (see Fig. 5.2). The ensembles were then classified based on  $L$  following the classification scheme in Sathe et al. [2011a]. For each stability condition, first, second (median) and third quartiles were computed. In the subsequent figures the first and the third quartiles represent the range of uncertainty of the respective turbulent statistic, whereas the second quartile denotes the median or approximately the ensemble average value. The choice of the second quartile is driven by the fact that the outliers do not influence ensemble statistics, which the mean value is normally influenced by (depending on the number of outliers). In the absence of too many outliers the mean and the median values are very close to each other.

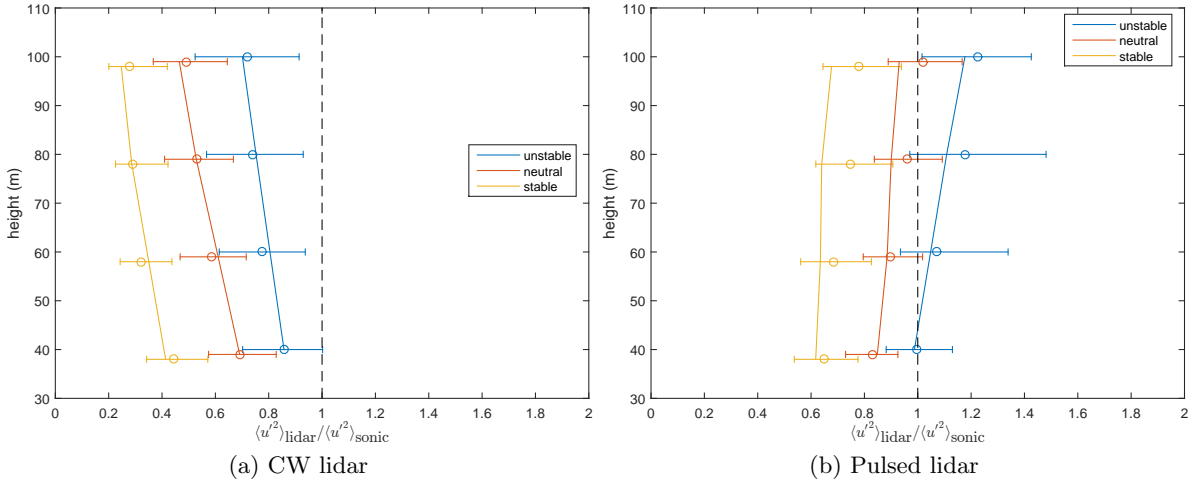


Figure 5.4: Ratio of the  $u$  variance estimated from the lidar measurements to that estimated by the sonics at different heights and atmospheric stabilities. The markers represent the measurements and the error bars represent the range determined by the first and the third quartiles. The solid lines represent model results. The plots are offset in heights for clarity.

Fig. 5.4 shows the ratio of  $\langle u'^2 \rangle$  estimated from lidar measurements to that estimated from the reference sonics for both types of lidars, i.e. a CW (Fig. 5.4a) and a pulsed lidar (Fig. 5.4b), at different heights and atmospheric stabilities. The measurements are shown by the markers and the error bars. The circles are the second quartiles and the error bars denote the range determined by the first and the third quartiles. Following the goal of this experimental campaign, a model was developed that attempted to understand the estimates of the components of  $\mathbf{R}$  by using the VAD method of processing the lidar data. Eq. (3.5) was used to compute the turbulence estimates from the model. The reference estimates from the model were computed using Eq. (1.7). Several inferences can be drawn from Fig. 5.4. First



and foremost it is clear that for any type of lidar a ratio of one at all heights and atmospheric stability would indicate that the turbulence estimates from the lidar measurements are as good as those from the reference sonics. Unfortunately that is mostly not the case. The ratio of turbulence estimates not only deviate from the ideal one but they are also very different for different types of lidars and are significantly influenced by the turbulence structure in the atmosphere.

For a CW lidar the ratio decreases with height and with increasing stability, i.e. from unstable to stable conditions. As we understand from chapter 3, a lidar never receives backscatter from exactly a point, but from all over the physical space, and mainly along the line-of-sight. The significant contribution is received from a certain Rayleigh length  $l$ , which for a CW lidar is approximated as being proportional to the distance at which measurements are obtained  $d_f^2$  [Sonnenschein and Horrigan, 1971](see also section 3.2.1.1). Clearly it shows that  $l$  increases quadratically with height, e.g.  $l$  at 80 m will be approximately four times that at 40 m above the ground. Simultaneously turbulence length scales do not increase quadratically with height, but approximately linearly [Peña et al., 2010]. As a consequence greater filtering of the turbulence signal occurs resulting in smaller estimations of turbulence statistics at higher heights than lower heights as compared to the reference sonics. Similarly turbulence scales are larger under unstable conditions than under stable conditions [Sathe et al., 2013], thereby resulting in smaller estimations of turbulence statistics under stable conditions than under unstable conditions. Fortunately the modelled behaviour of the estimation of  $\langle u'^2 \rangle$  from lidars and sonics correspond fairly well with the measurements, thereby increasing confidence in our understanding of turbulence estimations using the VAD method.

For a pulsed lidar the ratio increases with height but decreases with increasing stability. As opposed to a CW lidar,  $l$  remains constant with height. Considering that turbulence scales increase roughly linearly with height, it is obvious that larger estimates of turbulence statistics are observed at higher heights than at lower heights. The behaviour of the ratio under different atmospheric stabilities is similar to that observed for a CW lidar. However at some heights and atmospheric stabilities (mainly unstable conditions) the ratio is greater than one, which indicates that the estimates of  $\langle u'^2 \rangle$  are larger than those estimated from the reference sonics. This is explained by the cross contamination by the two-point correlations of the wind field components as is manifested by the Einstein summation in Eq. (3.5). As for the CW lidar the model estimates correspond fairly well with the measurements.

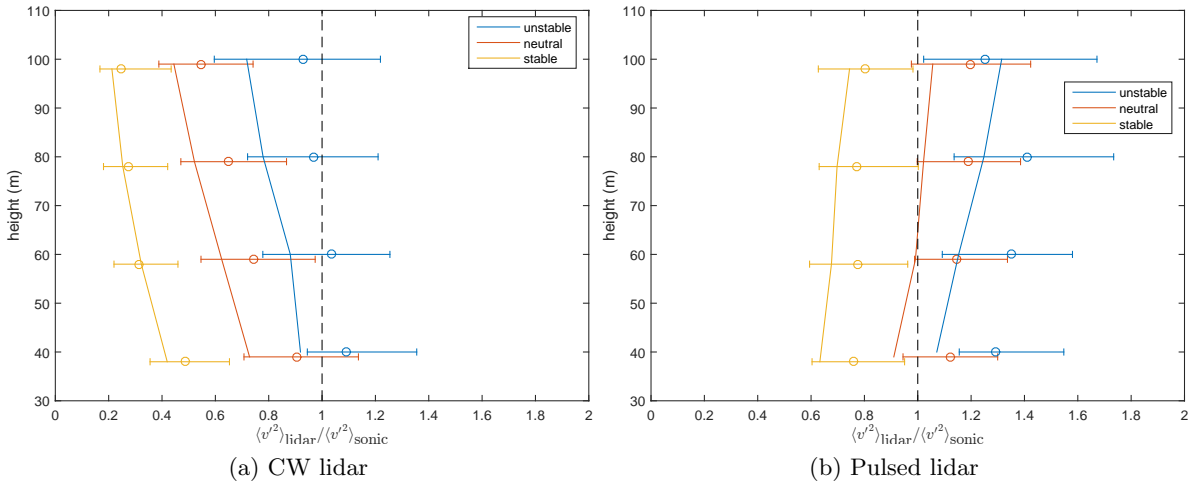


Figure 5.5: Same as Fig. 5.4, but for the  $v$  variance

Figures 5.5 and 5.6 show the ratio of  $\langle v'^2 \rangle$  and  $\langle w'^2 \rangle$  respectively estimated from lidar



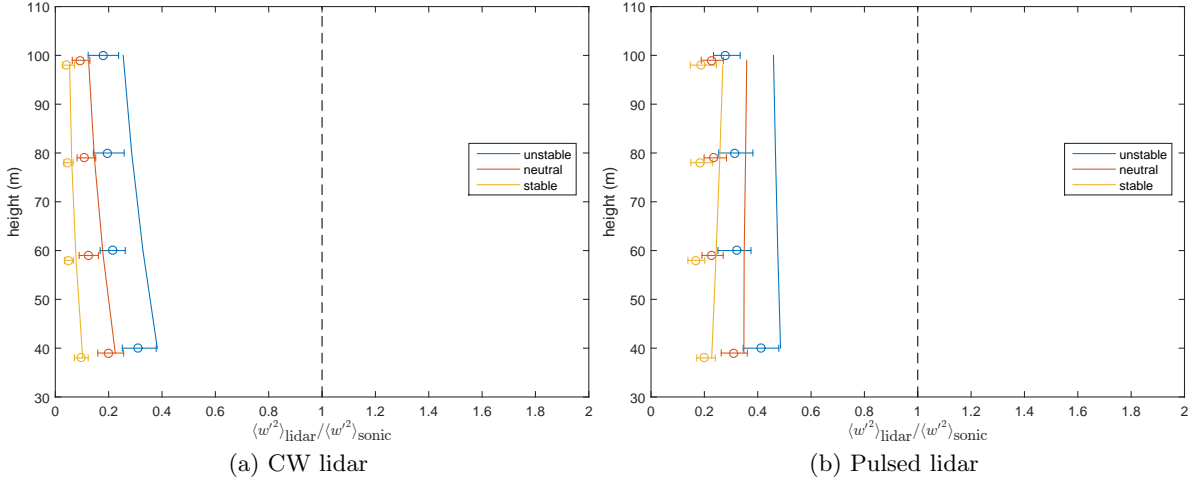


Figure 5.6: Same as Fig. 5.4, but for the  $w$  variance

measurements to that estimated from the reference sonics for both types of lidars. As for  $\langle u'^2 \rangle$  the behaviour of the ratio is similar for both types of lidars and under different atmospheric stabilities. There is however a significant difference in the magnitudes of the ratios between  $\langle w'^2 \rangle$  and  $\langle u'^2 \rangle$ . Figure 5.6a shows that turbulence estimates from the lidar measurements could be as small as only 10 % of the reference estimates under stable conditions for a CW lidar. This difference can be explained by the fact that the turbulence scales are significantly larger for the  $u$  component than for the  $w$  component (see Fig. 7 in Sathe et al. [2013]). Consequently the turbulence estimates are filtered significantly for the  $w$  component. For the  $v$  component there is not a significant difference in the magnitudes of the ratios, despite larger turbulence scales for the  $u$  component than for the  $v$  component as seen in Fig. 7 in Sathe et al. [2013]. On the contrary for  $\langle v'^2 \rangle$ , the ratios are slightly larger for both types of lidars than for  $\langle u'^2 \rangle$ . This could be due to the counteracting contribution of the cross-contamination as explained in section 3.1.1. The agreement between the model and measured estimates is slightly poorer than for  $\langle u'^2 \rangle$ .

## 5.2 Study 2

### 5.2.1 Introduction

The objective of this study was to understand the estimated turbulence spectra ( $F_{ij}(k_1)$ ) from the time series of a pulsed lidar, where VAD method was used to post process the lidar data. Modelling of the estimated  $F_{ij}(k_1)$  was carried out and comparisons were made with the measurements from a pulsed lidar. Modelled and measured  $F_{ij}(k_1)$  were also compared with the same estimated from reference sonics at two heights. For details on the model the reader is referred to [Sathe and Mann, 2012]. Owing to the fact that  $R_{ij}$  is simply an integration of  $F_{ij}(k_1)$ , this study complimented the study described in section 5.1 by further consolidating the understanding of the estimated  $R_{ij}$  from the lidar data using the VAD method.

### 5.2.2 Measurement details

The description of the site and the instrument details are exactly the same as those described in section 5.1.2 (refer to Figs. 5.1 and 5.2, and table 5.1). In this study only the pulsed lidar (WindCube) was used and investigations were performed at only two heights, i.e. 60 and 100 m. Furthermore the mean wind direction sector is chosen such that it is roughly aligned with

nominal E-W beams, i.e.  $130^\circ \leq \Theta \leq 140^\circ$ . The choice of the mean wind direction sector was driven by the fact that the nominal North beam of the lidar was  $45^\circ$  with respect to the true North. The  $u$  and  $w$  component measurements are then deduced from the nominal E-W beams and the  $v$  component is deduced from the nominal N-S beams.

The other criteria for the selection of the data are neutral atmospheric stability and a mean wind speed of 9 m/s. This wind speed was chosen because the Mann [1994] model parameters were available at 9 m/s. Practically the selection of the data is carried out with a mean wind speed in the interval 8-10 m/s, which resulted in 79 and 58 10-min time series of the sonics and the lidar at 60 and 100 m, respectively. Atmospheric stability is characterized using Obukhov length  $L$  (see Eq. 1.11) where the sonic measurements at 20 m were used to estimate  $L$ .

### 5.2.3 Mean wind speed comparisons

Since the period of measurement and the instruments used are the same as that stated in section 5.1.2, Fig. 5.3b is also representative for this study. Since only the mean wind speeds between 8 and 10 m/s are considered in this study, the reader should zoom in the respective wind speed interval to check the comparisons in Fig. 5.3b. Considering that the systematic error and the uncertainty in the mean wind speed estimation within the chosen interval is similar to that for the whole range of wind speeds, it gives enough confidence to proceed with the turbulence analysis.

### 5.2.4 Turbulence measurements

The high frequency lidar data of  $v_r$  are post processed using the VAD method (see section 3.1) to deduce the wind field components. Turbulence spectrum for each component is estimated using standard Fourier transformations [Pope, 2000]. Simultaneously the same are also estimated using the sonic measurements. The corresponding modelled estimations are carried out as described in Sathe and Mann [2012]. In the subsequent figures the spectra of the respective wind field components are denoted as  $F_u(k_1)$  for the  $u$  ( $i = 1, j = 1$ ) component,  $F_v(k_1)$  for the  $v$  ( $i = 2, j = 2$ ) component, and  $F_w(k_1)$  for the  $w$  ( $i = 3, j = 3$ ) component.

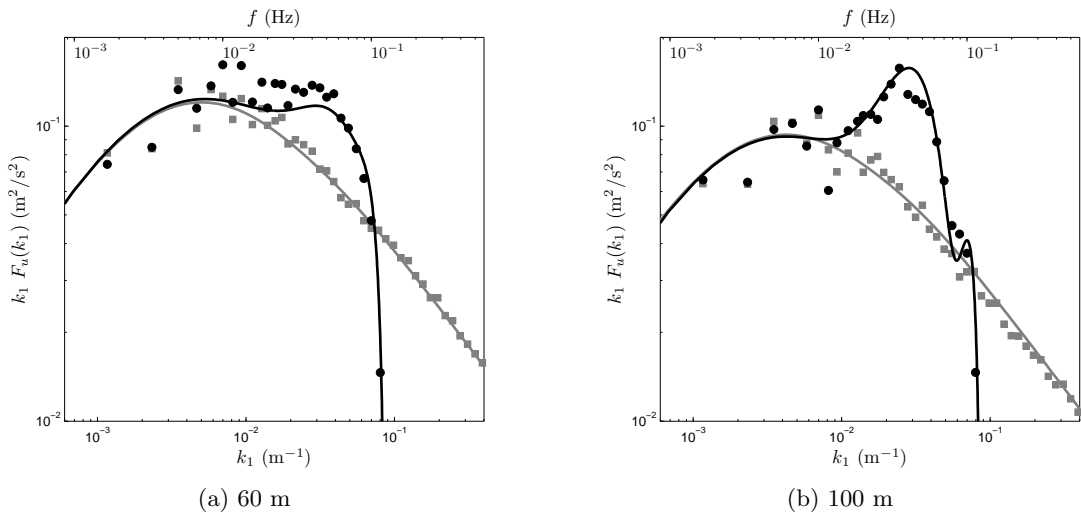


Figure 5.7: Comparison of the estimated  $u$  spectrum from the pulsed lidar (black) and reference sonics (gray) at 60 m and 100 m. The markers indicate measurements and the continuous line indicates the model.

Fig. 5.7 shows the comparison of the estimated  $u$  spectra from the pulsed lidar (black) and reference sonics (gray) at 60 and 100 m. The measurements indicate that the spectrum measured by the lidar deviates significantly from the standard surface-layer spectrum as the turbulence scales decrease approximately from  $k_1 > 0.005 \text{ m}^{-1}$ . Approximately in the inertial sub-range, where the sonic spectra scales with  $k_1^{-5/3}$ , there is an almost complete attenuation of the turbulence signal, and hence a rapid decrease in the spectral energy. This observation has a striking resemblance with that of Canadillas et al. [2010], where an independent measurement under neutral conditions in the German North Sea showed an increase in the spectral energy above  $k_1 > 0.005 \text{ m}^{-1}$  and subsequent rapid attenuation. One of the reasons for this redistribution of the spectral energy is the contribution of the two-point correlations between different components of the velocity field. At very low wavenumbers ( $< 0.005 \text{ m}^{-1}$ ), the spectral energy measured by the lidar is approximately the same as that measured by the sonics. This is because very large turbulence eddies are associated with very low wavenumbers that cause the volume measurement from the lidar to behave essentially like a point measurement.

At both heights, the model agrees very well with the measurements at almost all wavenumbers. The point-like behavior of the lidar at very low wavenumbers, and redistribution of the spectral energy beyond  $k_1 > 0.005 \text{ m}^{-1}$ , is captured fairly well. However, there are stark differences in the distribution of the spectral energy at 60 and 100 m. This is because of the beam interference phenomenon that occurs for certain separation distances at 100 m and is related to the assumption of Taylor's hypothesis [Sathe and Mann, 2012].

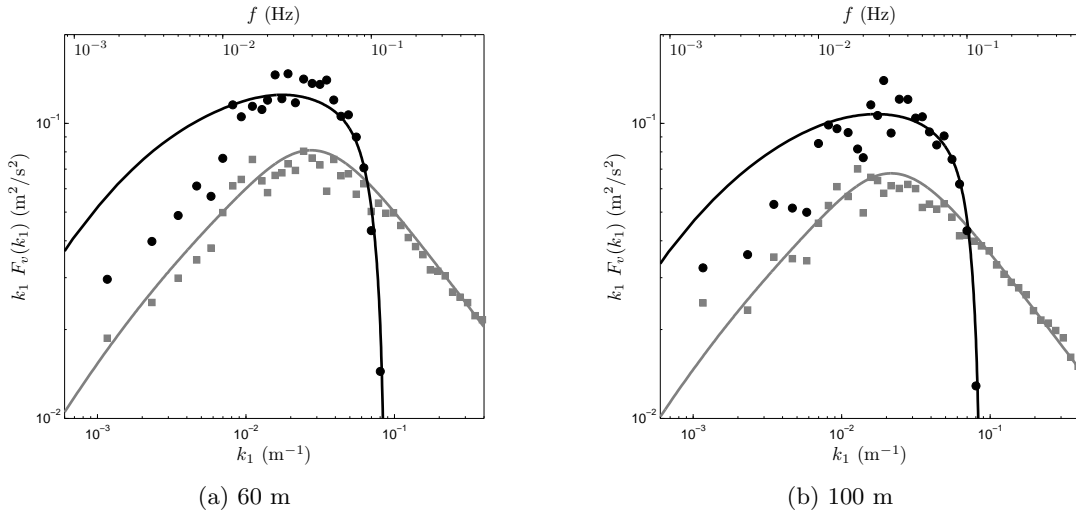


Figure 5.8: Same as Fig. 5.7 but for the  $v$  component

Fig. 5.7 shows the comparison of the estimated  $v$  spectra from the pulsed lidar (black) and reference sonics (gray) at 60 and 100 m. As observed for the  $u$  component, the  $v$  spectrum measured by the lidar deviates significantly from that of the reference sonic spectrum. However, at very low wavenumbers, there is an offset in the spectral energy between the lidar and the sonic. The behavior in the inertial sub-range is the same as that for the  $u$  component, where a rapid attenuation in the spectral energy is observed. The model agrees fairly well with the measurements at 60 and 100 m, except at very low wavenumbers ( $< 0.005 \text{ m}^{-1}$ ), where the model over estimates the spectral energy. One striking feature of this comparison is that as opposed to the  $u$  component, no beam interference phenomenon at 100 m is observed. This is because only those beams that are perpendicular to the mean wind direction (i.e. N-S beams) are used to deduce the  $v$  component. Thus, even though Taylor's hypothesis is assumed, the

beams never interfere with each other at any separation distance. [Sathe and Mann \[2012\]](#) provides a detailed explanation for the over estimation of the spectral energy at very low wavenumbers.

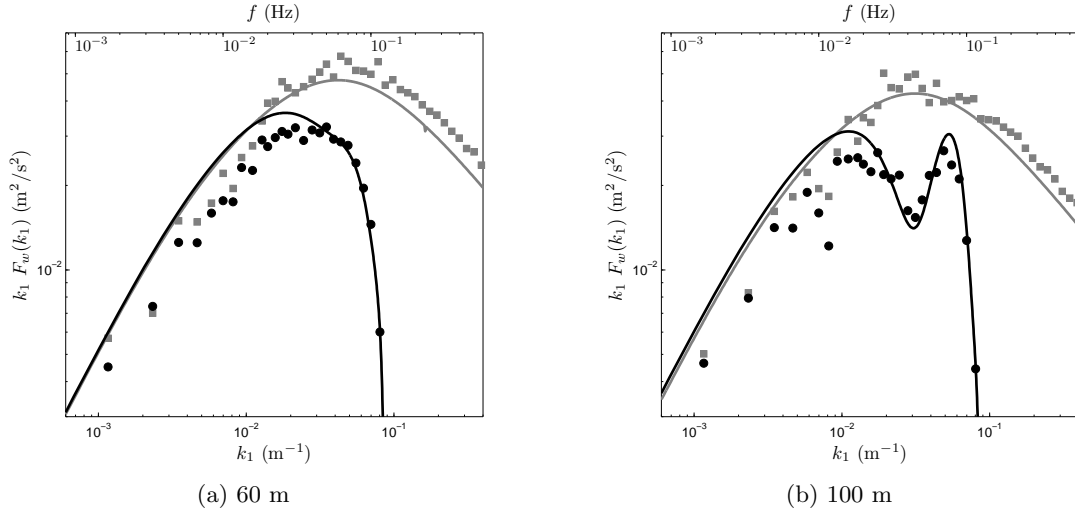


Figure 5.9: Same as Fig. 5.7 but for the  $w$  component

Fig. 5.7 shows the comparison of the estimated  $w$  spectra from the pulsed lidar (black) and reference sonics (gray) at 60 and 100 m. Similar to the  $u$  spectra, beam interference at 100 m is observed because of the assumption of Taylor's hypothesis, and because the same beams are used to estimate the  $w$  component as those used for the  $u$  component. The measured lidar spectrum agrees well with the model at both heights, especially at high wavenumbers. As observed for the  $u$  component, at 100 m the effect of unusual covariances on the spectral energies is also noted. At very low wavenumbers, there is a slight offset between the model and measurements. This offset could be because of the slight deviation in the modeled and measured sonic spectrum. The model also shows that at very low wavenumbers, because of very large turbulence eddies, the volume measurement from the lidar behaves similar to a point measurement.

## 5.3 Study 3

### 5.3.1 Introduction

The objective of this study was to demonstrate an alternative scanning method, the so-called six-beam scanning, where the estimation of the components of  $\mathbf{R}$  (see Eq. 1.2) is carried out using  $\langle v_r'^2 \rangle$  instead of the VAD method of post processing the lidar data. The motivation behind this study was to reduce the systematic errors that are otherwise observed to a very large extent in the VAD method of post processing the lidar data (see section 3.1.1 for theoretical summary and sections 5.1 and 5.2 for experimental evidence). For details of the six-beam method the reader is referred to [\[Sathe et al., 2015\]](#).

### 5.3.2 Measurement details

The details of the measurement site are exactly the same as described in section 5.1.2. Because the scanning strategy in this study is different from that used in the commercial WindCube lidar, a modified version of the lidar called as WindScanner was used that had the same pulsed

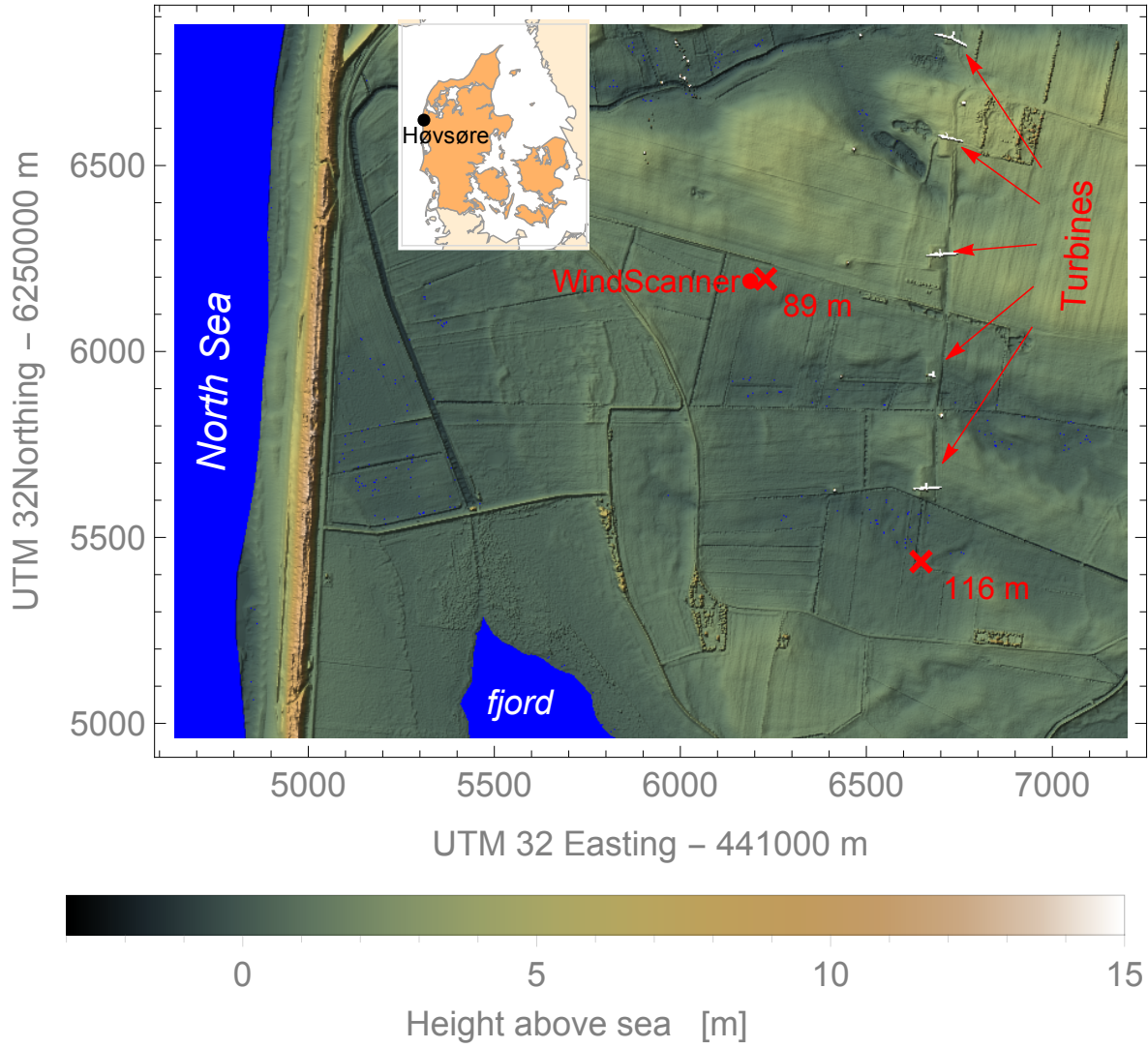


Figure 5.10: Location of the test site at Høvsøre, Denmark

lidar technology at the base. The WindScanner is based on the pulsed lidar Windcube 200 from Leosphere and a dual-axis mirror based steerable scanner head designed by DTU Wind Energy and IPU. The WindScanner is intended for radial velocity measurements from the range of distances between 50 and 6000 m. The current maximum measurement rate is 10 Hz. The maximum number of simultaneous radial velocities acquired at any rate along each line-of-sight is 500. The WindScanner can emit either 400 or 200 ns laser pulses, which are streamed with two corresponding pulse repetition frequencies of 10 and 20 kHz respectively. The energy content of 400 ns laser pulses is 100  $\mu\text{J}$ , while the energy content of the 200 ns laser pulses is half of this value. The scanner head has two rotational degrees of freedom and can rotate around the azimuth and elevation axes, thus it directs the laser pulses into the atmosphere at any combination of azimuth and elevation. The maximum scanner head rotation speed is 50  $^\circ\text{s}^{-1}$ , while the maximum acceleration is 100  $^\circ\text{s}^{-2}$ . The scanner head can rotate around both axes from 0 to 360  $^\circ$ , and the rotation can be endless. The pointing accuracy of the WindScanner is 0.05 $^\circ$ . The WindScanner is operated via a remote “master computer” through a UDP/IP and TCP/IP network using the remote sensing communication protocol (RSComPro) [Vasiljevic, 2014].

Figure 5.10 shows the location of the WindScanner and the 89 m met mast, which has

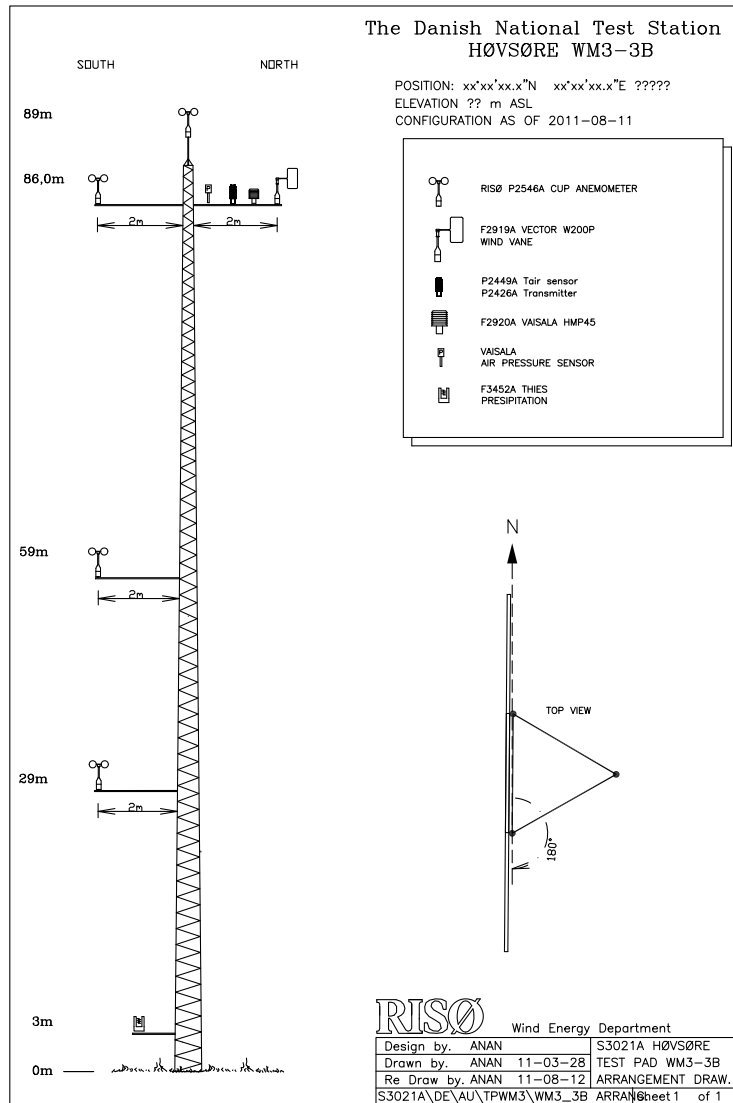


Figure 5.11: Sketch of the instruments on the 89 m met mast at Høvsøre, Denmark

a cup anemometer at the top and a wind vane at 86 m in the North direction (see Fig. 5.11). Since the wind turbines are to the east of the WindScanner and the met mast, the measurements only from the western sector ( $225-315^\circ$ ) are used. Owing to the sudden change in the surface roughness from sea to land in the western sector, we expect the turbulence structure to be influenced by the development of the internal boundary layer, particularly under different atmospheric stabilities. This presented a challenge in selecting a reference met mast. The two met masts are separated by a distance of about 850 m, which is of the same order as the distance between the WindScanner and the 116.5 m met mast. Initial comparisons of the 30-min mean wind speeds and turbulence statistics between the reference instruments on the two met masts indicated presence of horizontal inhomogeneity. Therefore the 89 m met mast was chosen as the reference, which is separated by a distance of approximately 41 m only from the WindScanner. Atmospheric stability was however characterized by an 80 m



Sonic on the 116.5 m met mast, and it was assumed that  $L$  is not significantly influenced by horizontal inhomogeneity.

Table 5.2: Instrument and Measurement Details of Study 3

	Reference Met Mast 1		Reference Met Mast 2	Pulsed Li-dar
	Cup Anemometer	Wind Vane	Sonics (only for estimating $L$ )	WindScanner
Location, UTM zone 32V WGS84 datum	447229 m, E and 6256195 m, N		447647 m, E and 6255435 m, N	447188 m, E and 6256189 m, N
Model/Version	Risø P2564A cup	F2919A Vector W200P	Metek USA1 F2901A	Windcube 200
Period of Measurement	July 1 – July 28, 2013			
Sampling rate (Hz)	10	10	20	$\approx 0.0667$
Averaging Period (min)	30			
Measurement Height (m)	89	86	80	89

Significant influence on the flow homogeneity in the horizontal direction around the scanning circle was also not expected, which is one of the key assumptions of the six-beam method. The measurements were compared to the reference cup anemometer placed at 89 m on the top of a met mast placed near the WindScanner. The duration of the full cycle of the six-beam measurements from the WindScanner was about 15 s. The turbulence statistics were estimated over an averaging period of 30-min. After filtering for data availability within each 30-min period, where only those periods were chosen with 95 % data, the number of 30 min periods reduced to 625. Finally filtering for wind directions to avoid wakes from the wind turbines and the met mast rendered 401 30-min periods. Table 5.2 provides details of the instruments used.

### 5.3.3 Mean wind speed comparisons

Fig. 5.12 shows the comparison of the mean wind speeds estimated the WindScanner and the reference cup anemometer at 89 m. It is observed that the systematic error (characterized by the slope of the linear curve fit), and the uncertainty (characterized by the coefficient of determination  $R^2$ ) are negligible. The closer the value of the slope to one, the smaller the systematic error, whereas the closer the value of  $R^2$  to one, the smaller the uncertainty. Since for both types of lidars the systematic errors and uncertainties with respect to a reference sonic are  $\lesssim 0.1\%$  only, it gives enough confidence to proceed with the turbulence analysis.

### 5.3.4 Turbulence measurements

From section 3.2.2, we understand that the optimum configuration of the six-beam scanning is as given in table 3.1. For each 30-min period  $\overline{v_r^2}$  was estimated using the time series of

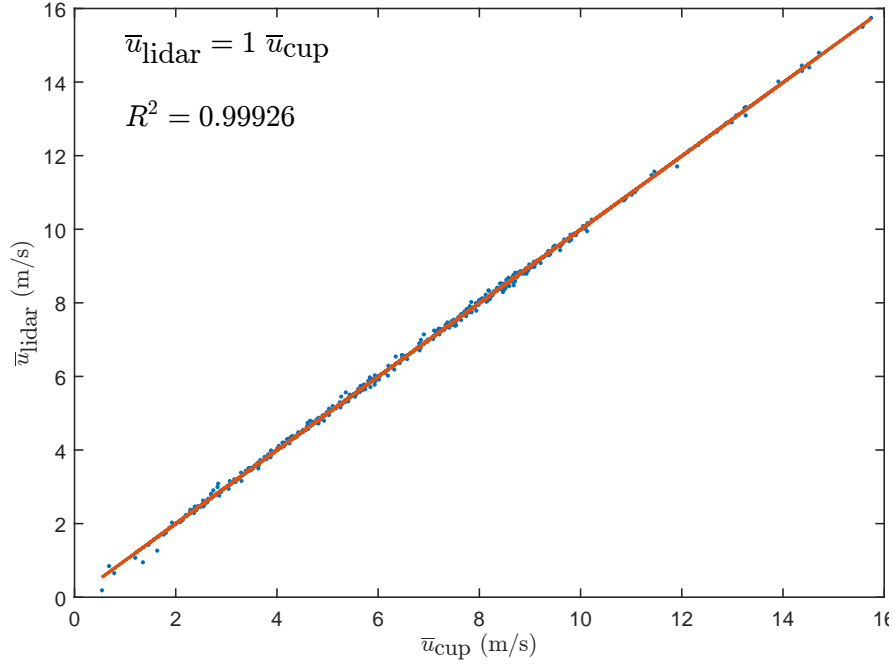


Figure 5.12: Scatter plot of the 30-min mean wind speeds estimated by the WindScanner and the reference cup anemometer at 89 m

$v_r$ . Using Eq. 3.14 the components of  $\mathbf{R}$  are estimated subsequently. In order to assess the performance of the method, the estimated statistics were compared with those estimated from the VAD method of post processing the lidar data. Finally the estimated  $\overline{u'^2}$  and  $\overline{v'^2}$  from both methods (six-beam and VAD) were compared with those estimated from the cup anemometer and vint vane data under different atmospheric stabilities (see table 5.2).

Figure 5.13 shows the comparison of the turbulence statistics derived from the WindScanner measurements using the six-beam and the VAD methods and those obtained from the cup anemometer under unstable conditions. It is clear that the six-beam method measures more turbulence, about 19 % for  $\overline{u'^2}$  and 3 % for  $\overline{v'^2}$  than the VAD method, where the orthogonal least-squares regression is used to fit the cup anemometer measurements. The scatter using both methods is comparable to each other, but there is a slightly more scatter using the six-beam method for  $\overline{v'^2}$ .

Figure 5.14 shows the same as Fig. 5.13 but under neutral conditions. As for the unstable conditions, the six-beam method measures more turbulence, about 18 % for  $\overline{u'^2}$  and 10 % for  $\overline{v'^2}$  than the VAD method. The scatter using both methods is comparable to each other, with the six-beam method giving a slightly reduced scatter than the VAD method.

Figure 5.15 shows the same as Fig. 5.13 but under stable conditions. As for the unstable conditions, the six-beam method measures more turbulence, about 19 % for  $\overline{u'^2}$  and 4 % for  $\overline{v'^2}$  than the VAD method. The scatter using both methods is comparable to each other, but there is a slightly more scatter using the six-beam method for  $\overline{u'^2}$ .

Thus under all stabilities the six-beam method is closer to the turbulence measurements carried out using the reference cup anemometer. There is however some probe volume averaging using both methods, but is significantly larger for the VAD method. The probe volume averaging can be observed clearly by comparing the radial velocity spectra, which can be observed in Fig. 6 in Mann et al. [2009], and Fig. 4 in Sjöholm et al. [2009]. From Figs. 5.13–5.15 it is clear that using both methods the WindScanner measures more turbulence under stable conditions than under unstable and neutral conditions. This may be contrary to our



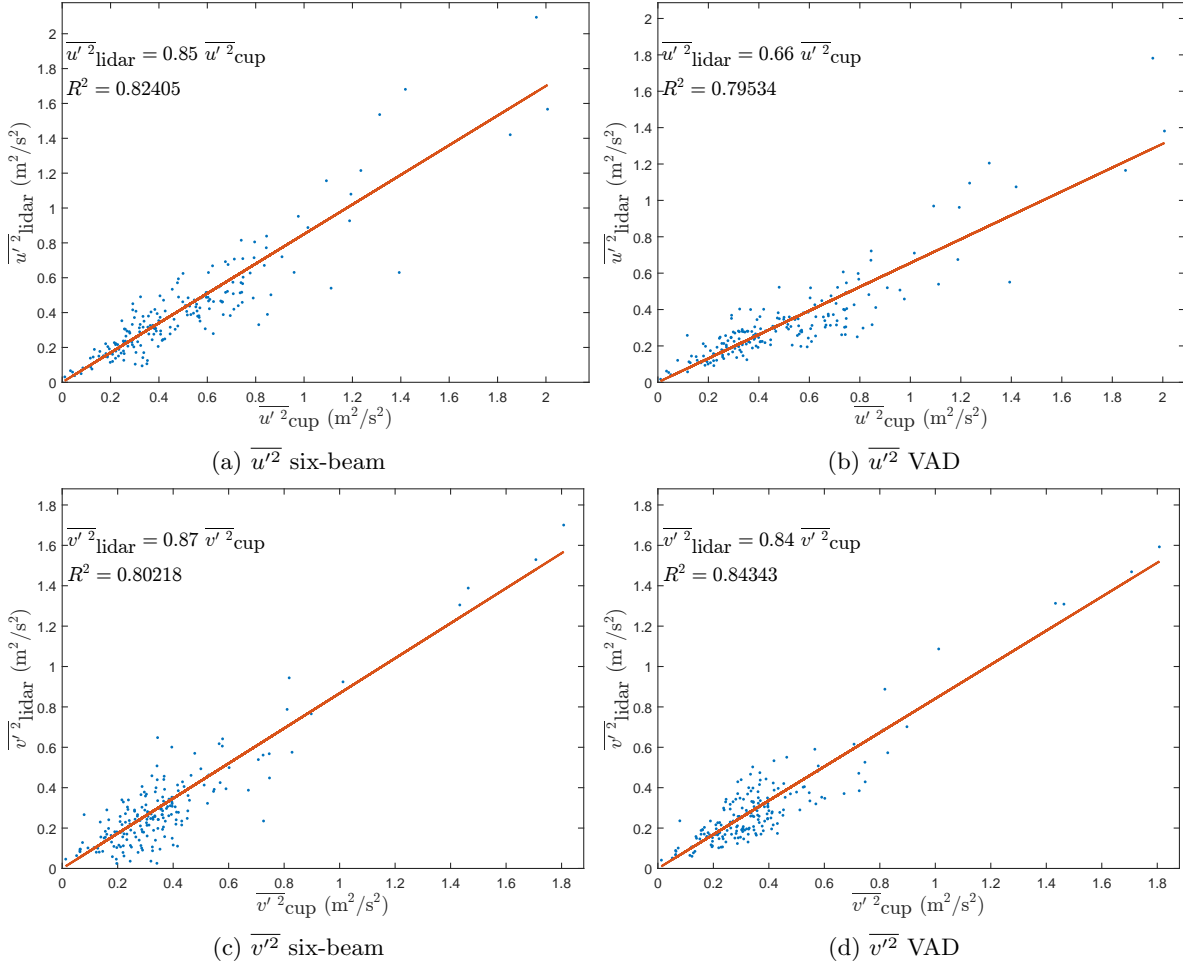


Figure 5.13: Comparison of the the turbulence statistics under unstable conditions between the WindScanner and the cup anemometer using two methods

intuitive understanding, because usually the turbulence scales are much larger under unstable conditions than under stable conditions [Sathe et al., 2013]. These results are also contrary to what has been observed by in study 1 (see Fig. 5.4–5.6) at the same site. However, it is to be noted that in study 1 only those lidar measurements were used when the wind was blowing from the eastern direction, whereas in this study only those measurements were used when the wind was blowing from the western direction (see section 5.3.2). As described in Sect. 5.3.2, in the western sector there is a sudden change of roughness due to the transition from sea to land. As a consequence there is a development of the internal boundary layer (IBL). Also the growth of the IBL depends on atmospheric stability, where under unstable conditions the growth will be faster than under stable conditions. Panofsky and Dutton [1984] state that the growth of the height of the boundary layer is proportional to the drag coefficient  $u_*/\bar{u}$ . And it is well known that the drag coefficient is larger for unstable stratification. Consequently the turbulence scales within the IBL will be smaller as compared to those outside of it. It is then interesting to check whether the WindScanner measures more within the IBL under unstable conditions as compared to the stable conditions.

Figure 5.16 shows the  $u$  and  $v$  spectra derived from high-frequency cup anemometer measurements under different stability conditions. If we define the characteristic length scale  $\mathcal{L}$  as the length scale corresponding to the maximum spectral energy, it is then clear that the peak of the  $v$  spectra is shifted to the right for unstable conditions as compared to the stable

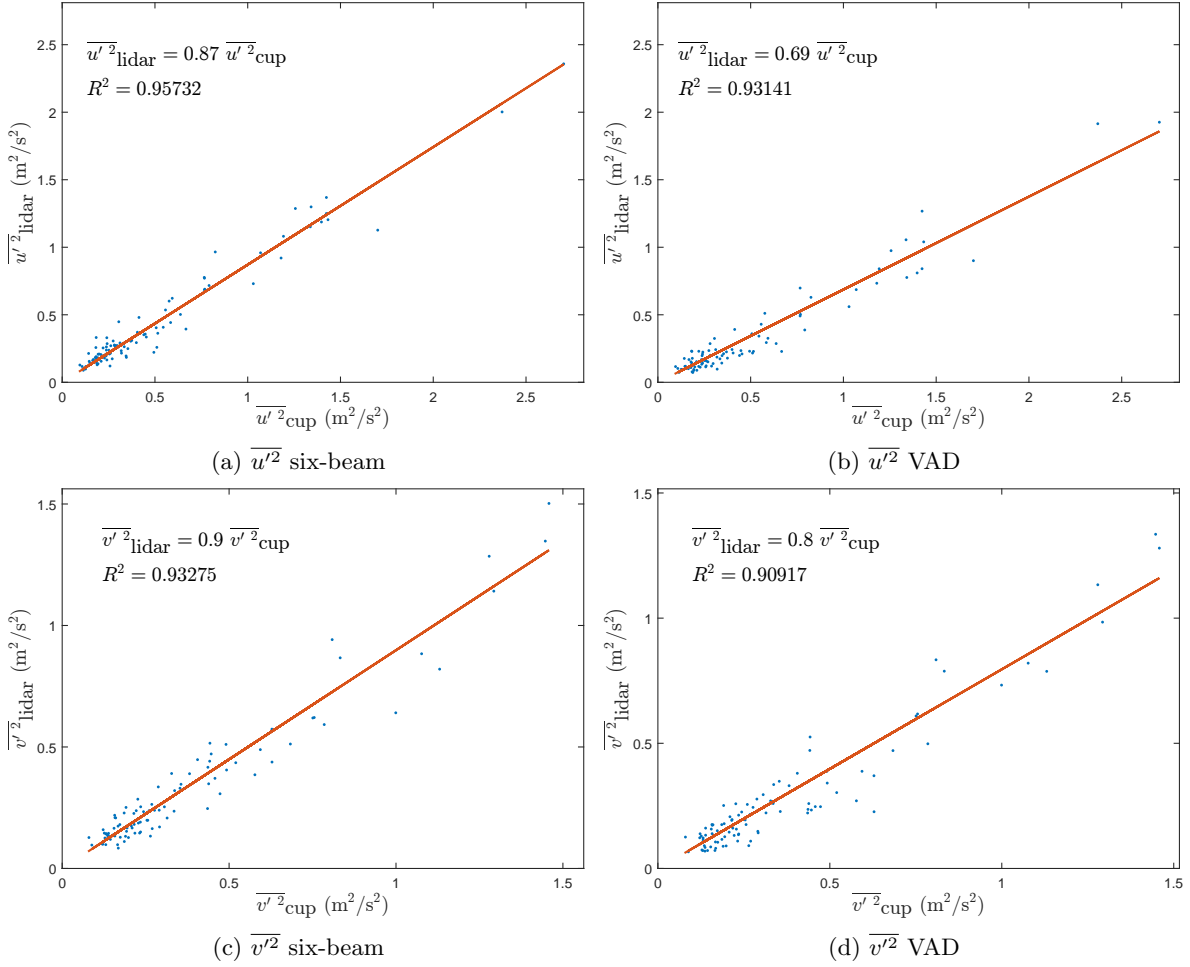


Figure 5.14: Same as in Fig. 5.13 but under neutral conditions

conditions. It is not that clear for the  $u$  spectra, however the shift of scales to larger wavenumbers under unstable conditions can still be observed. Thus  $\mathcal{L}$  appears smaller under unstable conditions than under stable conditions for the measurements from the western sector used in this work. There is thus more probe volume averaging under unstable conditions than under stable conditions. Hence the WindScanner attenuates the turbulence measurements more under unstable conditions than under stable conditions.

Another interesting observation is that using the VAD method the WindScanner does not measure more turbulence than the reference cup anemometer under any stability condition. This does not agree with that observed in study 1 (see Figs. 5.4b and 5.5b), even though the same basic pulsed commercial lidar technology was also used in that study. It is likely due to the fact that in study 1 only four beams were used as opposed to six beams in this study, and  $\phi$  was  $30^\circ$  compared to  $45^\circ$  used in this work. Therefore the turbulence statistics are not directly comparable with those obtained in study 1 even though the same basic commercial lidar was used. Due to the application of the least squares technique on the  $v_r$  measurements in this work, there is significant volume averaging around the scanning circle, which is also observed in study 1 for a continuous-wave lidar (see Fig. 5.4a, 5.5a and 5.6a).

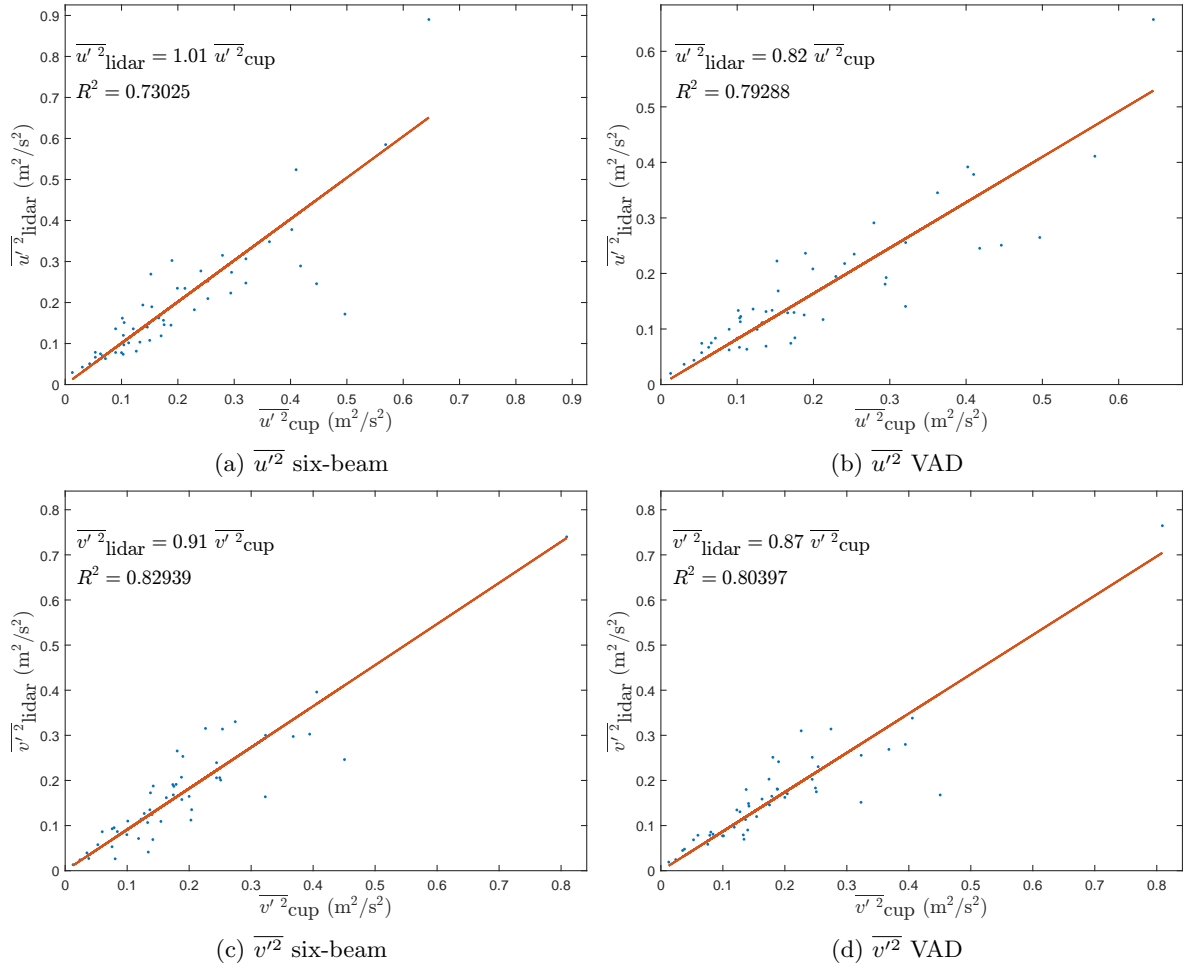


Figure 5.15: Same as in Fig. 5.13 but under stable conditions

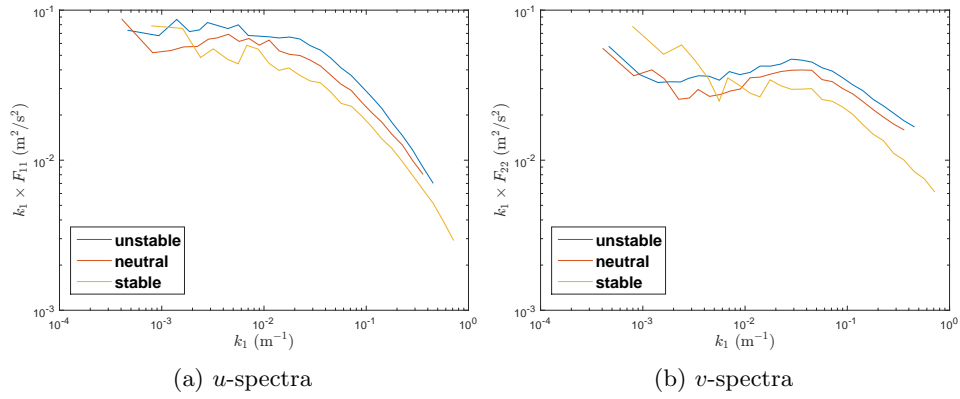


Figure 5.16: Comparison of the  $u$ - and  $v$ -spectra derived from high-frequency cup anemometer and wind vane measurements under different stability conditions

## Chapter 6

# Conclusions and Future Perspectives

### 6.1 Conclusions

At the outset it is to be noted that this report concerns characterization of atmospheric turbulence only on micro scales, and not on meso or synoptic scales. Following chapter 1, it is clear that usually, for meteorological or wind energy purposes, characterization of turbulence often requires estimating one or more turbulence quantities defined in Eqs. (1.1)–(1.12) at approximately a point. From Eq. (3.2) it is clear that the raw lidar measurements are never obtained at one single point, but over a certain probe volume. Thus unless the commercial lidars are used in a non-routine manner (see section 3.1.2), or novel techniques are used (see section 3.2), the smaller scales of turbulence, many of which are also relevant for wind turbines would always be filtered out, resulting in underestimation of a turbulence quantity in comparison to a reference instrument. It is also to be noted that the temporal resolution of lidar measurements is usually much lower than that of the reference instrument. While this will also result in filtering of smaller turbulence scales, its effect is not comparable unless the cycle time of one measurement is of the order of 30 s.

In the past decade, several commercial lidars, both CW and pulsed have sprung up, where the most common scanning strategy is placing the lidar on the ground and making conical scans (see section 3.1). In these lidars the routine method is to post-process the data using either the VAD or the DBS method to obtain the wind field components. From chapter 3, we also understand that apart from the aforementioned smaller scales averaging effect, this also results in contamination of the turbulence statistics due to the two-point correlations of different wind field components. As a consequence on some occasions we might obtain the right results for the wrong reasons. For example, in Fig. 5.4b, at 100 m, the ratio of  $\langle u'^2 \rangle$  estimated from the lidar measurements to that obtained from the reference sonic measurements is approximately one, which would erroneously indicate the ability of lidars to correctly quantify turbulence. However, as can be observed at other heights and atmospheric stabilities, the conflicting effects of the systematic errors, i.e. underestimation due to the probe-volume averaging, and overestimation due to the cross-contamination do not cancel each other. Therefore the ratio is other than one on most occasions. It is therefore recommended to not use the VAD or DBS method in estimating turbulence statistics.

Fortunately, as seen in chapter 4, significant research has been carried out in either post-processing the data better or in scanning configurations, so much so that even with a commercial lidar,  $\varepsilon$  could be estimated as described in section 3.1.2. The problem of contamination by cross-correlations can be reduced significantly by using the six-beam method with one lidar

(see sections 3.2.2 and 5.3), or by using a three lidar system (see section 3.2.5). The biggest challenge then is to recover the filtered smaller scales of turbulence, which could be carried out either by conically scanning lidars (see section 3.1.2) or by simply staring a lidar at one distance (see section 3.2.1). In principle one could then combine the retrieved large and small scale turbulence to characterize micro-scale turbulence.

A possibly important aspect of turbulence measurements using lidars that has not been considered in this report is the instrumental error, which is generally assumed to be uncorrelated [Frehlich et al., 1998, Lenschow et al., 2000]. Fortunately for modern commercial lidar systems, the magnitude of the instrumental error is not significant, and can be safely neglected [Mann et al., 2009]. However, for those lidar instruments that have significant instrumental error and therefore could potentially bias turbulence measurements, the techniques suggested by Frehlich et al. [1998], Drobinski et al. [2000] and Lenschow et al. [2000] can be used to perform corrections.

Finally it is encouraging to understand that the lidars themselves (commercial or research grade) do not exhibit any significant limitation in the technology. The routine methods of characterizing turbulence are however not recommended. Some additional tricks (as described in chapter 3) in either post-processing or scanning configurations are therefore required to obtain meaningful turbulence quantities. It is also recommended to perform as fast scans as possible in order to avoid filtering of smaller turbulence scales due to coarse temporal resolution.

## 6.2 Future Perspectives

Having seen in chapter 4 that a significant amount of research has been carried out to characterize atmospheric turbulence in a meaningful manner, an obvious question thus arises; is there anything new to be discovered with regards to processing raw lidar data, scanning configurations, or the technology itself that can provide more reliable turbulence measurements using lidars? The future perspectives are thus outlined:

1. Raw lidar data processing – up until now, the processing algorithms that have been developed have shown that by combining an isotropic turbulence model with lidar measurements, we are able to estimate  $\varepsilon$ ,  $\langle v_r'^2 \rangle$  and  $\mathcal{L}$  (see Table 4.1). However, turbulence is not isotropic in all range of scales. Anisotropy is particularly observed on longer length scales, and thus it is more desirable to estimate  $\langle v_r'^2 \rangle$  and  $\mathcal{L}$  by combining an anisotropic turbulence model [Kristensen et al., 1989, Mann, 1994] with the lidar measurements. This recommendation was also made by Frehlich et al. [2006] and Frehlich and Kelley [2008]. There is, however, a need for developing algorithms that make as little use of models as possible in combination with the measurements. Even an anisotropic turbulence model such as that created by Mann [1994] is based on a set of assumptions, e.g. neutral atmospheric conditions, applicability in the surface layer, validity of Taylor’s hypothesis, and it does not apply to complex terrain. If we then combine such a model with lidar measurements, and estimate turbulence parameters, then additional uncertainties may be introduced. In order to avoid such situations, further development of algorithms should also focus on making use of only raw lidar data to extract turbulence parameters, e.g., as shown in Mann et al. [2010].
2. Improvement in lidar technology – New, cheaper solid-state lasers for coherent detection lidars with integrated optical amplification are being developed and tested [Hansen and Pedersen, 2008, Rodrigo and Pedersen, 2008]. These may greatly expand the use of lidars for wind measurements, but they are not specifically tailored for turbulence

measurements. Preliminary tests of these lidars have been carried out by [Rodrigo and Pedersen \[2012\]](#), and show good comparison with a sonic anemometer. The solid-state lasers with integrated amplification may in the near future compete with the more expensive lasers used in C-W Doppler lidars. Direct detection is still on an experimental level [[McKay, 1998](#)] and has only been used in the atmosphere sporadically [[Dors et al., 2011](#), [Xia et al., 2007](#)]. The simple design of these instruments may eventually lead to cheaper lidar systems. Non-coherent detection may also provide possible new ways to estimate atmospheric turbulence [[Mayor et al., 2012](#), [Sela and Tsadka, 2011](#)], but to our knowledge it does not, so far, challenge the capabilities of the coherent Doppler lidars.

Completely new principles could also drastically improve the turbulence-measuring capabilities of lidars. One suggestion is to exploit the translation of the speckle pattern in the image plane of the lidar telescope. In this way, not only the line-of-sight velocity could be estimated, but also the two transverse velocity components. All components would be measured in the same volume, reducing the problem of cross-contamination. In the laboratory, this method has been successfully tested on translating hard targets [[Iversen et al., 2011](#), [Jakobsen et al., 2011](#)], but it is much harder to get the method to work with backscatter from atmospheric aerosols. Firstly, the return from the aerosols is much weaker and, secondly, the turbulence may reduce the correlation time of the speckle pattern, which could adversely affect the transverse velocity determination.

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DTU Wind Energy is a department of the Technical University of Denmark with a unique integration of research, education, innovation and public/private sector consulting in the field of wind energy. Our activities develop new opportunities and technology for the global and Danish exploitation of wind energy. Research focuses on key technical-scientific fields, which are central for the development, innovation and use of wind energy and provides the basis for advanced education at the education.

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